

# Investigation of the effect of using isotropic and anisotropic modelling techniques to study the cracking in masonry walls due to thermal movements of an overlying slab

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## Abstract

Use of roof slabs in Sri Lankan buildings is getting popular especially in commercial buildings and hotels, and even in some residential buildings. In a tropical country like Sri Lanka, roof slabs are exposed to direct solar radiation over more than eight hours per a day. Concrete slabs exposed to direct sunlight experience thermal movements. These movements can result in over stressing and cracking in underlying masonry walls. Due to the presence of vast range of geometrical and structural configurations, use of physical models to investigate the behaviour of masonry is a costly and a difficult exercise. As a solution, finite element modelling is widely used in investigation of masonry.

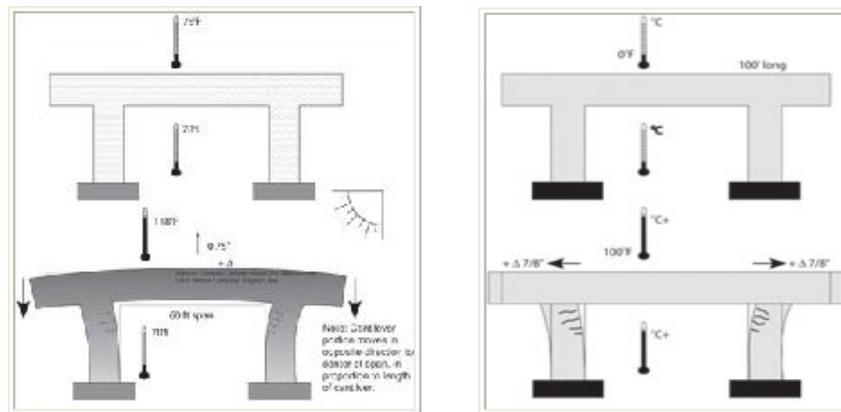
Masonry is a composite material that exhibits distinct directional properties due to the presence of mortar joints. Therefore, it is important to study the effect of the presence of these joints considering masonry as an anisotropic material rather than an isotropic material in numerical studies. This paper describes a numerical modelling exercise employed to understand the above effect related to the modeling of cracking in masonry walls due to thermal movements of an overlying slab. Further, models were used to study the effect of the aspect ratio, structural form and presence of other geometrical features such as openings and lintels on the above phenomenon. Also, the results were used to propose remedial measures to the problem. It was found that the pattern (type and location) of cracking depends significantly on the structural and geometric features of the wall and anisotropic approach can predict better results when structural and geometric features become complicated.

**Keywords:** anisotropic material, isotropic material, numerical modelling of masonry, principal stress, thermal cracking

# 1. Introduction

Cracking is the most common and visible defect found in masonry structures. The appearance of a crack is an indication of distress within the building. The cracking of masonry can be attributed to development of excessive stresses within the masonry and cracks are developed within the masonry once the internal stresses exceed the rupture strength of the material. The development of stresses can be due to external loads, movement of one or more parts of the structure, change of material or chemical action in the presence of moist air and water.

In addition to the warnings of structural concern, cracks can affect the value of the building, its marketability and insurability. On the other hand development of cracks can promote ingress of moisture into the walls. In this context appearance of cracks is unacceptable regardless of the reason for cracking, even if the cracks are not that structurally serious.



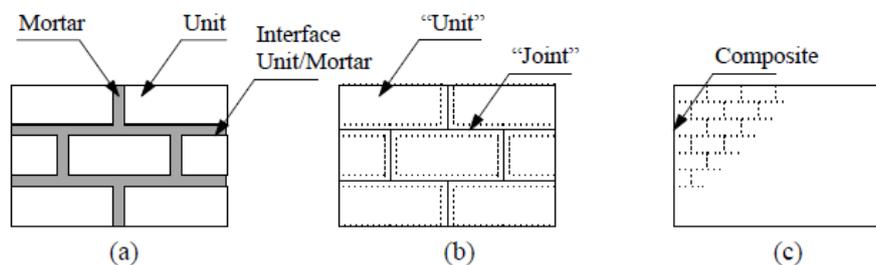
**Figure 1: Cracking of masonry wall due to thermal movements of an overlying slab (Dilrukshi & Dias 2008)**

Use of roof slabs in Sri Lankan buildings is getting popular especially in commercial buildings and hotels, and even in some residential buildings. In a tropical country like Sri Lanka, roof slabs are exposed to direct solar radiation over more than eight hours per a day. Concrete slabs exposed to direct sunlight experience temperature related horizontal movements. In addition, temperature of the top surface of the slab reaches higher temperature than the bottom, causing an upward deflection of the slab during heating. In a typical building, masonry and concrete elements restrain each other at their respective interfaces. Therefore, significant movement would be generated on the masonry walls, due to movement of the roof slab (see Figure 1).

These movements can result in overstressing and cracking in masonry. Therefore, in the buildings with roof slabs cracking of walls located immediately under the roof slab has become a considerable problem. Even though these cracks are non structural, they lead to considerable problems with respect to the performance and appearance of a building. In this context studying about these cracks and the factors causing these cracks is important to propose effective remedial/ preventive measures.

Due to the presence of a vast range of geometrical and structural configurations of masonry, use of physical models to investigate masonry is costly and difficult. As a result finite element method (FEM) has been widely used in the analysis of masonry structures and, various models have been developed to simulate the behaviour of masonry.

Masonry is a composite, heterogeneous, non-linear material that exhibits distinct directional properties as a result of the presence of mortar joints, which act as planes of weakness. Therefore, mechanical response of masonry is highly complex. Depending on the level of accuracy and simplicity desired, different modelling strategies have been used by masonry researchers and can be explained as Detailed micro-modelling, Simplified micro-modelling, and Macro modelling (See Figure 2).



**Figure 2: Modelling strategies for masonry structures: (a) detailed micro-modelling; (b) simplified micro-modelling; (c) macro-modelling (Lourenco, 1999)**

In the first approach units and mortar are represented by continuum elements while unit-mortar interface is represented by discontinuous elements. In the second approach, geometrically expanded continuum units are used, with discontinuous elements covering the combined behaviour of both mortar joints and interfaces. In this approach accuracy is lost compared with first approach, since Poisson's effect of the mortar is not included. In the third approach all three principal components of structural masonry are lumped into an equivalent continuum.

In investigations and studies of local behaviour of masonry micro modelling approaches are preferred, since both masonry and mortar can be modelled in detail. It had been used to model fracture of masonry by Rots (1991); Guinea et al. (2000); Bicanic, Stirling and Pearce (2000) and Mark, Peter and Robert (2004). However this approach is restricted to small test problems, due to the large amount of data and variables involved. In large and practice oriented analysis the knowledge of interaction between units and mortar is usually negligible for the global structural behaviour. In these cases macro modelling technique is extensively used (Rots 1997; Lourenco 1998; Dilrukshi and Dias 2008). In macro modelling, it is assumed that the size of the masonry structure is considerably larger than the units and mortar joints.

Most experimental determinations of strength values are based on uni-axial stress states. However in reality the problem involves at least a bi-axial stress state, if not a tri-axial stress state. It is known that strength in one direction is to some extent affected by the stress in the perpendicular direction. Therefore for better understanding of the material behaviour, biaxial or tri-axial strength is required (Samarasinghe 1980).

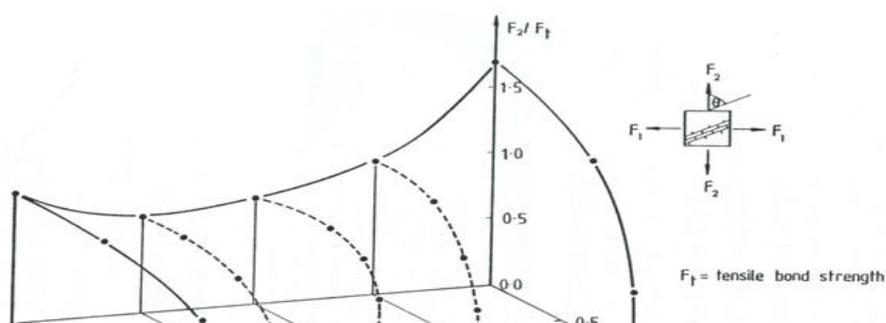
Because masonry is an anisotropic material, the elastic properties and strength characteristics will vary with the stress orientation relative to the bed. Therefore failure theories for isotropic materials cannot be directly applied for masonry, since in these failure theories the direction of stress has no significance on the strength properties of the material. Therefore failure of brick masonry cannot be defined simply in terms of principal stresses at any point. The influence of a third variable, the bed joint orientation relative to the principal stresses, must also be considered. Thus to completely define masonry failure a three-dimensional failure surface in terms of the principal stresses  $\sigma_1$  and  $\sigma_2$ , and their respective orientations to the bed joint  $\Theta$  and  $(90 + \Theta)$  is required. Hence, the stress state at a local failure point of a masonry wall subjected to in-plane loads would represent a particular point on this  $(\sigma_1, \sigma_2, \Theta)$  surface (Samarasinghe 1980).

Masonry exhibits distinct directional properties due to the presence of mortar joints. Depending upon the orientation of the bed joints to the applied stresses, failure can occur in the joints alone or in some form of combined mechanism involving the mortar and unit. The influence of bed joint orientation on the strength of masonry has been illustrated by many researchers (Allen 1973; Benjamin & Williams 1958; Chinwah 1972; Drysdale, Vanderkeyl & Hamid 1979; Page 1978; Shalin 1971; Yokel & Fattal 1976) by tests carried out with bed joints oriented at varying angles to the applied loading. The load carrying capacity and the failure mode were found to be critically depending on this angle. Therefore significance of anisotropic characteristic of masonry is evident.

There have been few attempts to obtain a general failure criterion for masonry subjected to in-plane loads due to the difficulty in developing representative biaxial tests and the large number of tests required. However the problem was quantitatively discussed by some researchers (Chinwah 1972; Yokel & Fattal 1976; Hendry 1978).

A failure envelope for brick masonry under bi-axial stress state was developed by Page in 1980. Biaxial tests were simulated using an iterative finite element programme which modelled bricks and joints separately and was capable of simulating a collapse mechanism after the progressive failure of a number of joints. The resulting failure surface (See Figure 3) is a function of principal stress ratio at any point and the inclination of the principal stresses to the bed joint.

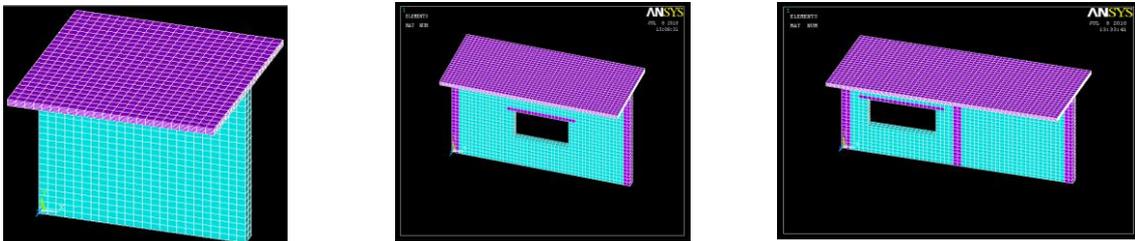
This paper explores the effect of anisotropic nature of masonry on the numerical modelling of cracking due to thermal movements of an overlying slab. Further this paper investigates the effect of the aspect ratio, structural form and presence of other geometrical features (such as openings and lintels) on the above phenomenal. Typical structural arrangements were mathematical modelled using the commercially available software ANSYS 11.0. Finally this paper illustrate on possible remedial measures based on the analysis of results.



*Figure 3: Biaxial stress failure envelope developed by Page (1980)*

## 2. Methodology

A typical masonry wall connected to the tributary slab area (See Figure 4) was considered, rather than analysing a whole building. Continuity of the slab was modelled using boundary conditions; slab was restrained for the translation in the direction perpendicular to the plane of wall. Since the objective of the study was to investigate the overall behaviour of the masonry wall, masonry was modelled as one constituent rather than modelling bricks and joints separately; macro modelling approach was used. Details of the assemblies used for the study are given in Table 1.



*Figure 4: Few idealised assemblies used in the finite element model*

In all of the models walls were considered to be having a thickness of 250 mm and a height of 3m. For each configuration of wall assembly, two separate models were developed; one considering masonry as an isotropic material and the other considering masonry as an anisotropic material. Concrete was considered as homogeneous and isotropic in all the models. Analysis was carried out to investigate the effect of structural forms (i.e. the effect of wall being load bearing or infill) and the aspect ratio (walls of lengths 3 m, 6m and 9 m were used to investigate the effect of aspect ratio of wall). Further analysis was extended to investigate the masonry cracking behaviour with the presence of complex geometrical and structural features such as openings and lintels. An opening of dimensions 2 m x 1 m was considered in this case with the presence of lintel extending up to 250 mm away from the edges of the opening (which is referred to as short lintel). A long lintel extending throughout the length of wall was also employed in some of the assemblies to investigate whether it would provide a shelter against the cracking (however was found to be ineffective after the study). The other remedial measure investigated was to employ a physical separation of the wall from the concrete frame.

**Table 1: Details of models considered for the study**

Model	Structural Type	Length	Opening	Lintel	Separation
M1/MA1	LB	3m	-	-	-
M2/MA2	LB	6m	-	-	-
M3/MA3	LB	9m	-	-	-
M4/MA4	FR-1	3m	-	-	-
M5/MA5	FR-1	3m	√	Small	-
M6/MA6	FR-1	3m	√	Long	-
M7/MA7	FR-1	3m	√	Small	√
M8/MA8	FR-1	6m	-	-	-
M9/MA9	FR-1	6m	√	Small	-
M10/MA10	FR-1	6m	√	Long	-
M11/MA11	FR-1	6m	√	Small	√
M12/MA12	FR-2	6m	-	-	-
M13/MA13	FR-2	6m	√	Small	-
M14/MA14	FR-2	6m	√	Small	√
M15/MA15	FR-2	9m	-	-	-
M16/MA16	FR-2	9m	√	Small	-

(M1 to M16- isotropic models, MA1 to MA16- anisotropic models, LB- Load bearing walls, FR1-One bay concrete framed walls, FR2-Two bay concrete framed walls)

Concrete-masonry interface was treated as completely bonded at in all the models considered. Three dimensional eight noded iso-parametric elements SOLID65 and SOLID 45 were employed in modelling concrete and masonry elements respectively. Both masonry and concrete was modelled as linear elastic materials. The material property constants- modulus of elasticity (E), poisson's ratio ( $\nu$ ) and co-efficient of thermal expansion ( $\alpha$ ) was used to define the stress-strain relationship of the materials. The material properties (See Table 2) were taken from earlier studies done on numerical modelling of cracking in masonry (Dilrukshi & Dias 2008). When the masonry was considered as an anisotropic material the elastic modulus of masonry in the direction perpendicular to joints was considered as the half the value in the other directions as recommended in literature (Lourenco 1997; Rots 1997). Shear modulus in each direction was calculated accordingly using the appropriate value of modulus of elasticity and poisson's ratio.

**Table 2: Material properties**

	Concrete	Masonry
Weight per unit volume ( $\text{kN/m}^3$ )	23.6	20

Modulus of elasticity (kN/m <sup>2</sup> )	25x10 <sup>6</sup>	1x10 <sup>6</sup>
Poisson ratio	0.2	0.2
Coefficient of thermal expansion	9.9x10 <sup>-6</sup>	6x10 <sup>-6</sup>

The tensile and shear strength of masonry was considered as 0.2 N/mm<sup>2</sup> and 0.13 N/mm<sup>2</sup> respectively. Thermal load on roof slab was assigned based on the maximum temperature gradients reported in the previous studies by Dilrukshi & Dias 2008. Accordingly thermal load on slab was assigned by dividing the slab in to six iso-thermal layers, having the temperatures 60 °C, 54.19 °C, 48.74 °C, 43.77 °C, 39.25 °C and 35 °C (from top to bottom).

Locations and directions of possible cracks were identified using the maximum principal stress values and their orientations obtained from the mathematical models (See Figure 5), based on the bi-axial failure envelope developed by Page 1980 (See Figure 3).

### 3. Results and Discussion

A summary of the results of all assemblies investigated is shown in Table 3.

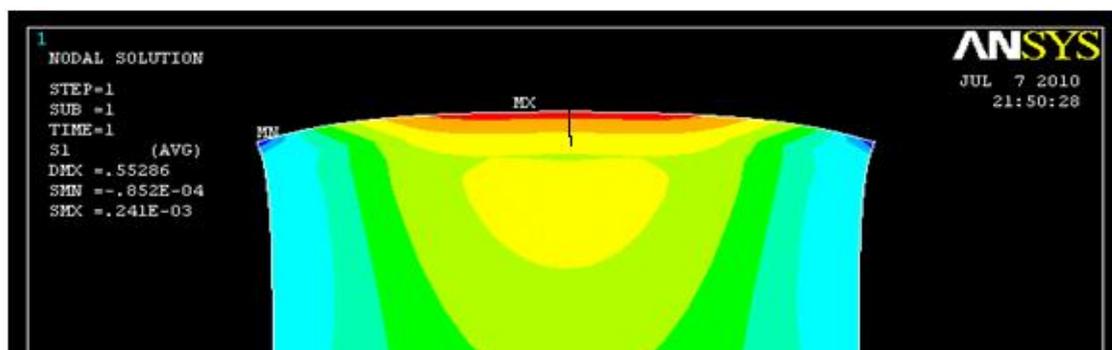
#### 3.1 Load-bearing Walls

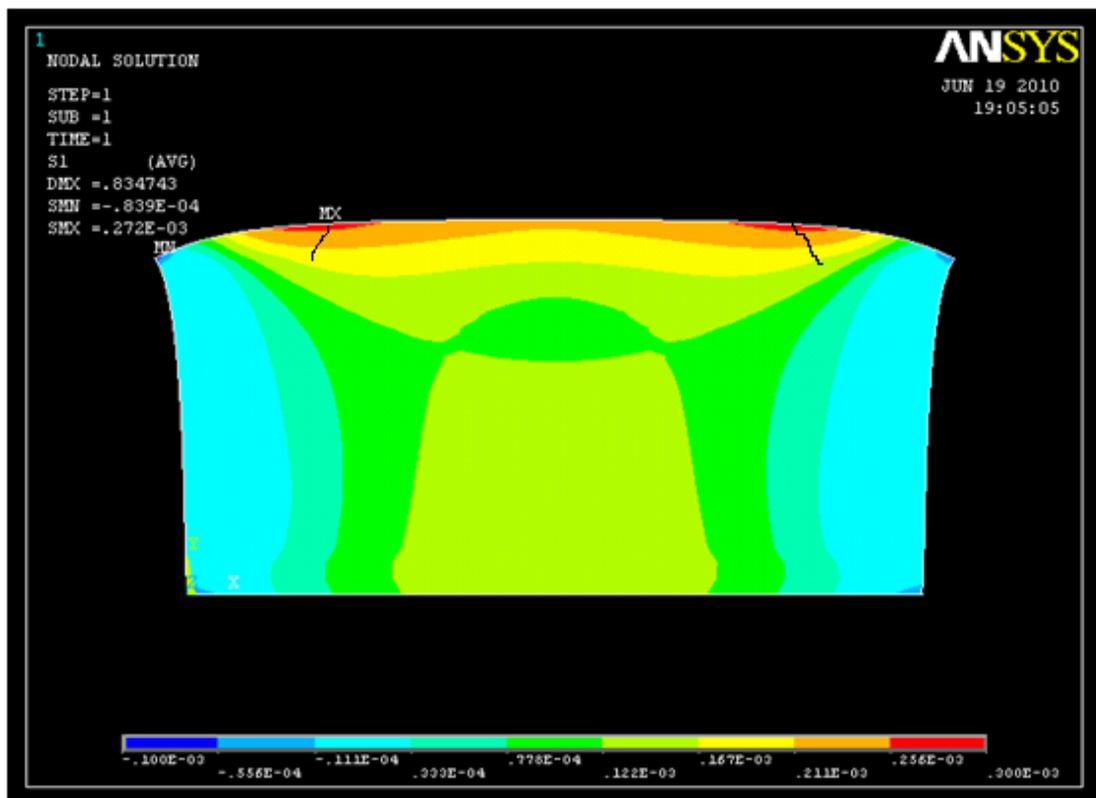
In load bearing walls all the assemblies considered are vulnerable to cracking regardless of the aspect ratio of wall. In 3 m load bearing wall propagation of a vertical crack at the soffit level closer to the centre of the wall is possible both in the isotropic (M1) and anisotropic (MA1) models. Resulting stress patterns are more or less same in M1 and MA1.

In 6 m load bearing walls (M2 & MA2) two inclined cracks (25<sup>0</sup>-30<sup>0</sup>) closer to two outer edges of the wall is observed. Resulting stress contours are more or less similar for M2 and MA2, except that the crack inclination and the resulting maximum principal stress (about 7% less) is slightly less in MA2 compared to M2.

In 9 m load bearing walls (M3/MA3), two inclined cracks (30<sup>0</sup>-35<sup>0</sup>) closer to the edges of the wall can be expected. Stress patterns are similar in both M3 and MA3, except that crack inclination and maximum principal stress (about 10% less) is slightly less in anisotropic model (MA3) compared to isotropic model (M3).

Resulting maximum principal stress values are higher in the walls with a higher aspect ratio (i.e stresses in 9 m wall > stresses in 6 m wall > stresses in 3 m wall). Inclinations of cracks are also followed the same pattern. In general, for the load-bearing walls resulting stress patterns of corresponding isotropic and anisotropic models are similar.





*Figure 5: Principal stress contour diagrams generated through the mathematical models developed for M1 and M2*

*Table 3: Summary of results*

Model	Closer to beam	Around opening

	Max. Principal Stress N/mm <sup>2</sup>	Crack inclination to vertical (deg)	Location (from outer edge), m	Max. Principal Stress	Crack inclination to vertical (deg)
M1	0.24	0	1.50	-	-
MA1	0.23	0	1.50	-	-
M2	0.27	31	1.25	-	-
MA2	0.25	25	1.375	-	-
M3	0.31	34	1.375	-	-
MA3	0.28	30	1.5	-	-
M4	N.C.	-	-	-	-
MA4	N.C.	-	-	-	-
M5	0.24	52	0.125	0.48	80
MA5	0.21	44	0.125	0.28	78
M6	0.25	52	0	0.53	77
MA6	0.23	42	0	0.32	73
M7	N.C.	-	-	N.C.	-
MA7	N.C.	-	-	N.C.	-
M8	N.C.	42	1.625	-	-
MA8	N.C.	-	-	-	-
M9	0.2	75	1.00	0.61	82
MA9	N.C.	-	-	0.39	82
M10	0.21	81	1.00	0.60	80
MA10	N.C.	-	-	0.40	79
M11	N.C.	-	-	N.C.	-
MA11	N.C.	-	-	N.C.	-
M12	0.32	20	3	-	-
MA12	0.34	20	3	-	-
M13	0.36	26	3	0.64	71
MA13	0.37	24	3	0.30	68
M14	N.C.	-	-	0.26	-
MA14	N.C.	-	-	N.C.	-
M15	0.31	42	1.375	-	-
MA15	0.33	21	4.5	-	-
M16	0.33	23	4.5	0.92	71
MA16	0.35	22	4.5	0.69	67

(N.C. – No Cracking, i.e. it refers to the instances where the stresses generated are inadequate to develop cracks)

## 3.2 Concrete-framed Walls

In short concrete framed walls (aspect ratio  $<1$ ) the resulting stress levels are not adequate to cause cracking in isotropic as well as in anisotropic models considered.

In 6 m single-panel concrete framed wall (for isotropic case), the stress levels are marginally adequate to cause cracking (M8); two diagonal cracks at a distance of about 1.625 m from the edge of the wall can be expected. In the corresponding anisotropic model (MA8), the resulting stress levels are not adequate to cause cracking.

In 6 m two-bay concrete framed wall (M12 and MA12) two inclined cracks (with an inclination of about  $20^\circ$ ) at the inner top corner of each panel can be expected. Stress patterns are more or less similar in isotropic and anisotropic models. In 9 m two-bay concrete framed wall (M15 and MA15), two inclined cracks closer to the edge of the panel can be expected.

In general, in single-bay concrete framed walls, the resulting stresses are lower than that of corresponding load bearing walls. In short single-bay walls, stresses developed are not adequate to cause cracking. With the increasing length, wall is marginally stressed to cause two inclined cracks close to the perimeter of the wall. In two-bay concrete framed walls the stresses are higher than that of corresponding load bearing walls and one-bay framed walls. Inclined cracks at the panel edge are possible in this case.

In load bearing walls and single-bay framed walls, the resulting stress levels of anisotropic walls are slightly lower than that of isotropic walls. In two-bay framed walls, the stresses of anisotropic walls are slightly higher than that of isotropic walls. However in general, stresses are more or less similar in corresponding isotropic and anisotropic models.

## 3.3 Walls with Openings

In walls with openings, the location most vulnerable to cracking is the top corner edges of the opening. In the case of 3 m wall with an opening for isotropic condition (M5), maximum principal stress of  $0.48 \text{ N/mm}^2$  occur at the two top corners of the opening. This maximum principal stress value is almost 200% of the value of that in 3 m solid load-bearing wall and about 240% of the value of that in 3 m solid concrete-framed wall. It can be expected two symmetric cracks about  $80^\circ$  inclined to the vertical at the top corner edges of the opening (in M5). In the corresponding anisotropic model (MA5), the resulting maximum principal stress value is less than that of M5 (which is about 50% less). However the location and inclination of cracks are similar to that of M5.

In 6 m single-bay concrete framed wall with an opening (for isotropic condition), the resulting maximum principal stress is about  $0.61 \text{ N/mm}^2$  (M9) which is about 275% of that value of 6 m wall without an opening (M8). In M9, two symmetrical cracks (with an inclination about  $82^\circ$ ) at the top corner edges of the opening can be expected. The resulting crack locations and

inclinations in MA9 is similar to M9, except that the maximum principal stress value is about 35% lower than M9.

In 6 m two-bay framed wall with an opening (M13), the resulting maximum principal stress is  $0.64 \text{ N/mm}^2$ . Maximum principal stress occurs at the inner top corner edge of the opening, regardless of cracking at both of the top corner edges as in the single-panel assemblies (M5, MA5, M9, and MA9), due to the asymmetry of the assembly in M13. Therefore a single crack with an inclination of  $71^\circ$  can be expected at the inner top corner edge (top corner edge closer to the middle column) of the opening. In the corresponding anisotropic assembly, a crack at the inner top corner edge of the opening can be expected. The only differences are that the crack inclination ( $68^\circ$ ) is slightly lesser than that of M13 ( $71^\circ$ ) and the resulting maximum principal stress is about 53% less than that of M13.

In 9 m two-bay concrete framed wall with a central opening (M16), the resulting maximum principal stress value is about  $0.92 \text{ N/mm}^2$  which is higher than all the assemblies considered in this study. In this case a crack with an inclination of about  $71^\circ$  at the inner top corner edge of the opening can be expected as the initial crack where as in the corresponding anisotropic assembly (MA16), a crack with an inclination of  $67^\circ$  at the inner top corner edge of the opening can be expected. The differences between the assemblies M16 and MA16 are that, in MA16 crack inclination ( $67^\circ$ ) is slightly less than that of M16 ( $71^\circ$ ) and the maximum principal stress in MA16 ( $0.69 \text{ N/mm}^2$ ) is about 25% less than that of M16.

When above results are considered in overall, it can be seen that the walls with openings exhibit much higher stresses than the walls without openings in all cases. In the case of single-panel walls, two symmetrical inclined cracks at the two upper corners of the opening is expected to form, while a single crack at the top corner of the opening closer to the inner column is expected in the case of two-panel concrete framed walls. With the increasing aspect ratio of the wall, stress levels get increases in both walls with openings and walls without openings.

Unlike in load bearing and concrete framed solid walls, in the walls with openings, resulting stress levels of anisotropic models are very much lower than that of isotropic models (maximum principal stress of anisotropic model is about 25 to 55% less than that of corresponding isotropic model). This deviation of stress value between isotropic and anisotropic models is higher for shorter walls compared to longer walls.

### **3.4 Remedial Measures**

In this study, two types of remedial measures were investigated; a long lintel extending throughout the length of the wall and physical separation of the wall from the frame.

Based on the results of models with a long lintel, it was observed that the resulting stress values are similar to the models with a short lintel. Therefore a long lintel is ineffective as a remedial measure.

In models with a physical separation of wall from frame except in M14, (i.e. M7, MA7, M11, MA11, MA14) the resulting stress values are significantly low and are inadequate to cause cracking. Even in M14, physical separation of wall has contributed to reduce the maximum principal stress by more than 50%. Based on these results it is evident that physically separating the wall from the concrete-frame at the wall-beam interface and wall-column interface (up to a height about one-third of the wall) is an effective remedial measure to avoid or to minimise the possibility of cracking of walls due to thermal movement of an overlying slab.

## 4. Conclusions

1. The pattern of cracking significantly depends on the structural form, aspect ratio and on the presence of other geometrical features such as lintels and openings.
2. For walls without openings (both load-bearing and concrete-framed walls) the resulting stress patterns are more or less same whether masonry is modeled as an isotropic material or anisotropic material.
3. For the walls with openings the resulting stresses in anisotropic model are considerably lower than the corresponding isotropic model.
4. Separating the wall from the concrete frame at wall-beam interface and wall-column interface is an effective solution for the problem (in concrete framed walls).

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