

## A SIMPLE FRACTURE CRITERION TO PREDICT FAILURE OF STEEL STRUCTURES IN EXTREMELY-LOW CYCLE FATIGUE REGION

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**Abstract:** This paper presents a simple criterion to predict the failure of steel structures due to the interaction effect of fracture and fatigue which is termed as extremely-low cycle fatigue (ELCF) failure. The criterion has been obtained from further simplification of available cyclic void growth model (CVGM). Initially the simplified ELCF fracture criterion is clearly presented and associated ELCF fracture prediction methodology is also indicated. The simplified criterion is then employed to determine ELCF fracture of some structural models. Hence verification of the simplified criterion is confirmed by comparing the results with previous criterion-based estimations. Then the simplified criterion is applied to predict the ELCF fracture of a reduced beam section specimen. Finally, study tends to conclude that the simplified criterion produces reasonable accurate prediction to ELCF fracture of steel structures where magnitude of triaxiality remains relatively constant.

**Keywords:** Low cycle fatigue, Earthquake loading, Cyclic plasticity, Finite element method

### 1. Introduction

The mechanism of extremely-low cycle fatigue (ELCF) was recently recognized with some of sudden failures of existing structures, which were characterized by large scale cyclic yielding due to occasional loadings such as earthquakes, typhoons. Generally, experimental approaches are popular for ELCF failure prediction. As for the authors view, only one theoretical study has recently been published [1], and the observed failure mechanism is based on void growth process. The fracture is calculated to occur when cyclic void growth index ( $VGI_{cyclic}$ ) exceeds its critical value. The  $VGI_{cyclic}$  demand is calculated based on complex integrations of a function, which depends on triaxiality and incremental plastic strain. However, it is required to modify commonly available finite element method (FEM) employed programs to cater this integration and finally it hindered the usage of general propose FEM packages as it is to estimate fracture in ELCF region. As a result, found applications of this criterion are very less.

Therefore, this study tends to simplify the above criterion to provide a new criterion to assess the real ELCF fracture of steel structures using available general-purpose FEM packages. However,

application of this criterion is limited to the situation where triaxiality remains relatively constant during its loading history. As highlighted from previous studies [2, 3], in many realistic situations, this statement can be applicable. The failure mechanism of this criterion is also similar as previous criterion. But the fracture criterion is totally different from the previous one such that the ELCF fracture is calculated to occur when accumulated equivalent plastic strain at cyclic loading exceeds its critical value. Initially, the details of simplified criterion are briefly indicated. Then the fractures of several models in ELCF region are estimated. Hence verification of the simplified criterion is confirmed by comparing the results with previous criterion-based estimations. Finally, study tends to conclude that the proposed criterion gives reasonably accurate prediction to fracture in ELCF region of steel structures where triaxiality remains relatively constant during its loading.

## 2. Simplified ELCF fracture criterion

The simplified fracture criterion and associated failure mechanism are briefly summarized in this section. For further understanding of the details about simplification of this criterion from the previous CVGM model and there distinguish features, refer authors' previous publications [4, 5].

The failure mechanism of this criterion mainly depend on two main aspects such as level of void growth (demand), including the effects of void growth and shrinkage during reversed cyclic loading and critical level of the void growth (capacity), related to cyclic strain concentrations of the inter void ligament material. Once this void growth demand is reach to its capacity the failure is determined. The situation where magnitude of triaxiality remains relatively constant during the loading history, level of void growth is properly described by accumulated equivalent plastic strain [4, 5]. Hence the simplified ELCF fracture criterion is defined as,

$$\bar{\epsilon}_p^{cyclic} > (\bar{\epsilon}_p^{cyclic})_{critical}$$

(1)

where  $\bar{\epsilon}_p^{cyclic}$  is the accumulated equivalent plastic strain at cyclic loading and it can be determined by subtracting equivalent plastic strain that has accumulated during every tensile excursion of cyclic loading ( $\bar{\epsilon}_p^t$ ) from accumulated equivalent plastic strain for every compressive excursion of cyclic loading ( $\bar{\epsilon}_p^c$ ) as follows.

$$\bar{\epsilon}_p^{cyclic} = (\bar{\epsilon}_p^t - \bar{\epsilon}_p^c)$$

(2)

The critical value (capacity) of accumulated equivalent plastic strain at cyclic loading  $(\bar{\epsilon}_p^{cyclic})_{critical}$  can be obtained as a degraded function of critical value (capacity) of accumulated equivalent plastic strain at monotonic loading  $(\bar{\epsilon}_p^{monotonic})_{critical}$  as bellow.

$$(\bar{\epsilon}_p^{cyclic})_{critical} = (\bar{\epsilon}_p^{monotonic})_{critical} \exp(-\lambda \epsilon_p^{accumulated})$$

(3)

where the  $(\bar{\epsilon}_p^{monotonic})_{critical}$  is critical value (capacity) of accumulated equivalent plastic strain at monotonic loading. Also this parameter represents the critical level of void growth (critical void size) under monotonic loading and it is determined by following expression.

$$(\bar{\epsilon}_p^{monotonic})_{critical} = \alpha \exp(-1.5T)$$

(4)

The  $\alpha$  is termed as toughness index which is an experimentally determined material constant. The  $\epsilon_p^{accumulated}$  in Eq. (3) is defined as the equivalent plastic strain that has accumulated up to the beginning of each tensile excursion of loading. As described by Eq. (3), the critical accumulated equivalent plastic strain at cyclic loading  $(\bar{\epsilon}_p^{cyclic})_{critical}$  reduces to critical monotonic limit for

monotonic loading situation. Because the accumulated equivalent plastic strain  $\epsilon_p^{accumulated}$  is zero for monotonic loading situations since it is calculated at the beginning of each tensile excursion. Hence the increment of cumulative equivalent plastic strain during current tensile cycle is not contributing to the damage that occurs within that loading increment. As a result the damage level is at a constant value within each tensile excursion. However, the accumulated equivalent plastic strain during current tensile cycles contributes only to void growth process, such that for each tensile cycle  $\bar{\epsilon}_p^{cyclic}$  is compared to a constant value of  $(\bar{\epsilon}_p^{cyclic})_{critical}$  which is calculated at the beginning of that cycle. This explanation confirms that ELCF fracture can only be initiated during tensile loading excursions.

The prediction of ELCF crack initiation is made when the  $\bar{\epsilon}_p^{cyclic}$  exceeds its critical value  $(\bar{\epsilon}_p^{cyclic})_{critical}$  over a characteristic length measure ( $l^*$ ) in the region of high stresses and plastic strains. However, places where triaxiality varies significantly during loading histories, the mentioned simplified criterion might produce a less accurate result than the CVGM criterion.

The described ELCF fracture criterion involve three parameters such as toughness index ( $\alpha$ ), damageability parameter ( $\lambda$ ) and the characteristic length ( $l^*$ ). The  $\alpha$  and the  $\lambda$  are determined through testing and finite element analysis of circumferentially smooth-notched tensile specimen. The characteristic length ( $l^*$ ) can be determined through the micro-structural measurements and observation of the fracture surface [1].

To utilize this fracture criterion, it is needed to determine the mentioned plastic variables during the multiaxial cyclic loading. Here, it is compulsory to perform a proper elasto-plastic analysis using a proper cyclic hardening model, which is compatible to complex structures with a reasonable accuracy.

### 3. Verification of simplified ELCF fracture criterion

Simplified ELCF fracture criterion is verified by comparing the simplified criterion predicted fracture displacements with CVGM [1] predicted results of three different structural models. In this comparison, a single hardening model is utilized with both fracture criteria [6, 7]. The A572-grade 50 steel is considered as the constructed material of all four models. The ELCF material constants, toughness index ( $\alpha$ ), damageability coefficient ( $\lambda$ ) and characteristic length ( $l^*$ ) are 1.18, 0.49 and 0.18 mm respectively [1]. This material exhibits nearly non-linear kinematic hardening behavior.

#### 3.1 Fracture prediction of a plate with a hole (Model 1)

The geometry of the considered structural model is shown in Figure 1. Considering symmetry of the geometry, loading and boundary conditions, the one-fourth of the geometry was subjected to FE analysis. The nine-node isoperimetric shell element was used for FE mesh as shown in Figure 3. Initially considered geometry is subjected to the monotonic load analysis. By observing the stress

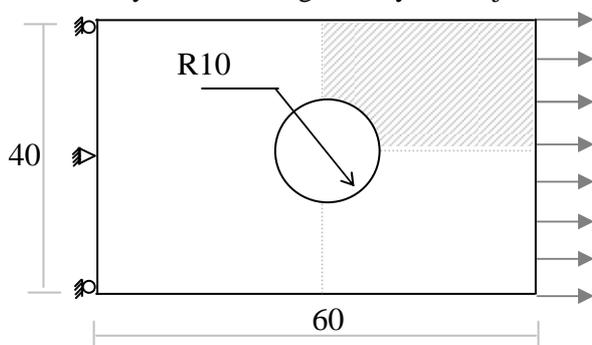


Figure 1: Geometrical details of Model 1

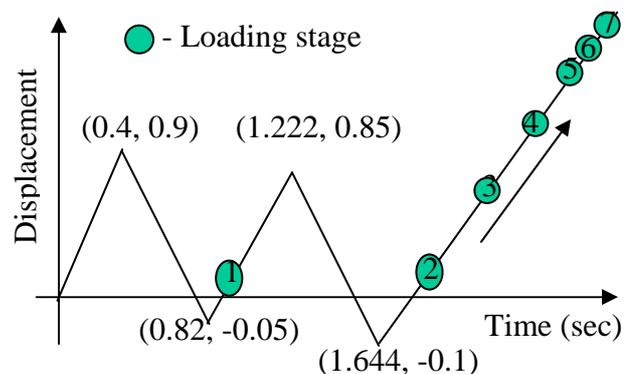


Figure 2: Displacement history of Model 1

distribution at ductile fracture (stress contour is shown in Figure 4), it is able to conclude that the critical zone lies along the transverse centerline of the specimen as shown in Figure 4. From the monotonic load analysis, variation of triaxiality ( $T$ ) versus effective plastic strain was plotted for sampling Gauss points at critical zone (Figure 5). These variations reveal that the triaxiality  $T$  does not illustrate significant variation with increment of plastic loading and the average value was considered for future calculations. Hence, the critical values of accumulated equivalent plastic strain

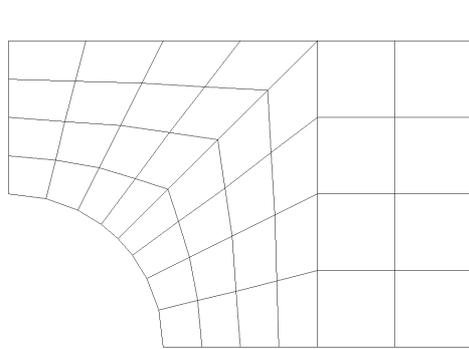


Figure 3: FE mesh for Model 1

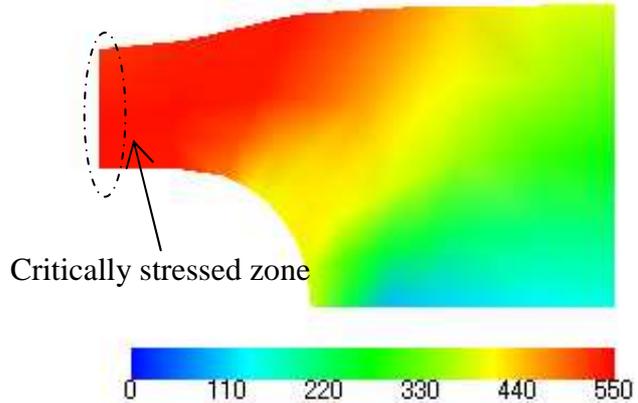


Figure 4: von Mises stress contour of Model 1

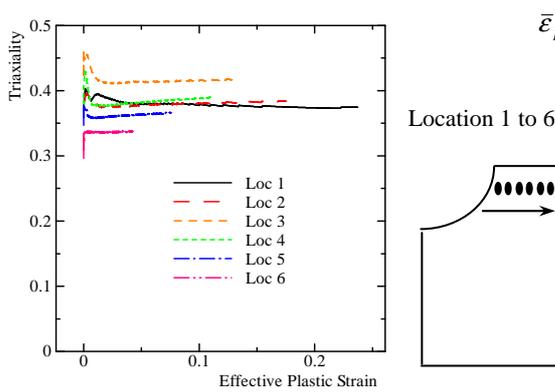


Figure 5: Triaxiality variation versus plastic strain at different location for Model 1

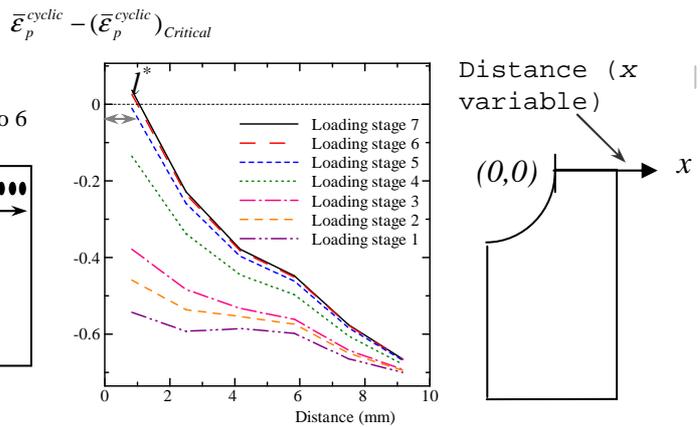


Figure 6: The  $\bar{\epsilon}_p^{cyclic} - (\bar{\epsilon}_p^{cyclic})_{Critical}$  variation along the length of critical zone for Model 1

$((\bar{\epsilon}_p^{monotonic})_{critical})$  at monotonic loading are calculated for each sampling Gauss points along the transverse centerline of the model. Then FE elasto-plastic analysis was conducted for cyclic loading based on considered hardening model. The applied displacement history is indicated in Figure 2. Finally, cyclically degraded values of the critical accumulated equivalent plastic strain,  $(\bar{\epsilon}_p^{cyclic})_{critical}$  variations are plotted at different loading stages. Simultaneously, the demands of accumulated equivalent plastic strain are also calculated for sampling Gauss points. Hence  $\bar{\epsilon}_p^{cyclic} - (\bar{\epsilon}_p^{cyclic})_{critical}$  variations along the transverse centerline were determined as shown in Figure 6. Finally, ELCF macro crack initiation was made when  $\bar{\epsilon}_p^{cyclic} - (\bar{\epsilon}_p^{cyclic})_{critical}$  exceeds zero over the characteristic length at loading stage seven. The applied displacement corresponding to this loading stage is recorded as the fracture displacement in Table 1.

### 3.2 Fracture prediction of a hollow cylindrical pier (Model 2)

The corresponding geometric details are shown in Figure 7. The horizontally applied uni-directional ground displacement is considered for this model as shown in Figure 8. The whole geometry was subjected to FE analysis using nine-node isoperimetric shell element. The followed procedures for ELCF fracture prediction are similar to the case of Model 1. The displacements at fracture are recorded in Table 1.

### 3.2 Fracture prediction of a hollow squared pier (Model 3)

The geometric details are shown in Figure 9. In this case also the horizontally applied uni-directional ground displacement is considered for this model as shown in Figure 10. The FE analysis was conducted using nine-node isoperimetric shell element. The followed procedures for ELCF prediction

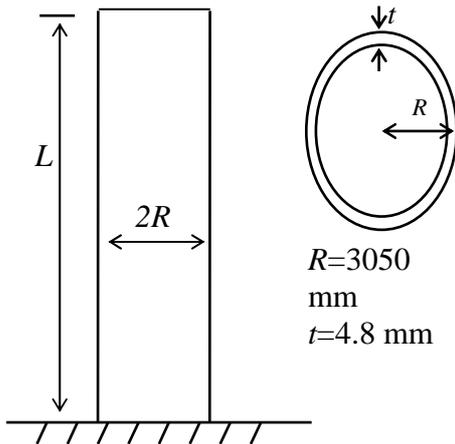


Figure 7: Geometrical details of Model 2

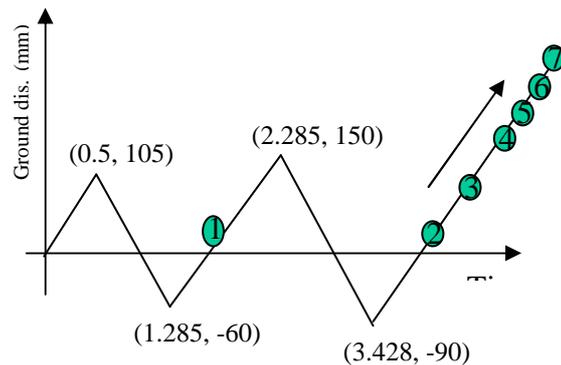


Figure 8: Displacement history of Model 2

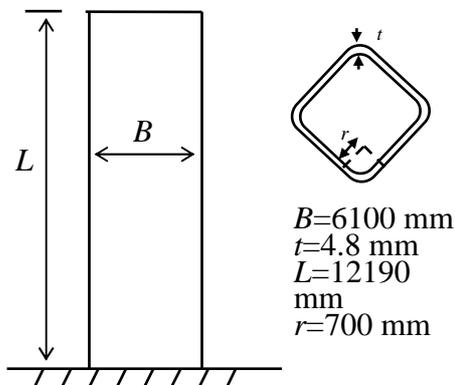


Figure 9: Geometrical details of Model 3

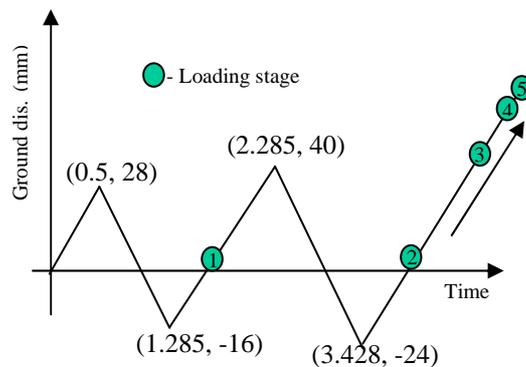


Figure 10: Displacement history of Model 3

are also as same as Model 1. The Table 1 shows the applied displacements of ELCF fracture.

Table 1: Comparison of ELCF fracture displacement of structural models

Model	Description	ELCF fracture displacement (mm)		Difference (%)
		CVGM Criterion	Simplified Criterion	
1	Plate with a hole	2.43	2.35	3.4
2	Hollow cylindrical pier	190.20	213.30	10.8
3	Hollow squared pier	65.62	62.05	5.8

Comparison of fracture displacements of both criteria in Table 1 shows that there is a similarity between fracture criteria. However, the percentage wise difference of fracture displacement of Model 2 is slightly higher than other models. Assumed reason is that, even though the Triaxiality ( $T$ ) of the Model 2 is relatively constant, it shows slight rate of increment during plastic loading. However, this comparison reveals that simplified ELCF fracture criterion produces reasonable accurate predictions to structures where the Triaxiality is relatively constant.

#### 4. Case Study: Fracture prediction of a reduced beam section specimen

During the Northridge and Kobe earthquake, welded connections in steel moment frames were found to be a weak link in structural systems [1, 2]. After these disasters, reduced beam section specimen (RBS) or dog-bone type connection detail (Figure 11) was developed to concentrate the plastic hinge a certain distance away from the connection within the beam. Though such connections have the potential to prevent sudden and brittle failures such as those in welded connections, there is always the possibility of ductile fracture under large plastic strains. This section initially describes the experimental determination procedure of ELCF fracture of RBS specimen. Then theoretical ELCF fracture was also determined for same RBS specimen. Finally the accuracy and the applicability of simplified fracture criterion are verified by performing comparisons to obtained results.



Figure 11: *RBS type connection*

Considering symmetry of RBS specimen (Figure 12-(a)) geometry, loading and boundary conditions, the one-fourth of the geometry was subjected to FE analysis. The nine-node shell element was used for FE mesh as shown in Figure 13-(a).

Initially considered geometry is subjected to the monotonic load analysis. By observing the stress distribution at ductile failure (stress contour is shown in Figure 13-(b)) it is able to conclude that

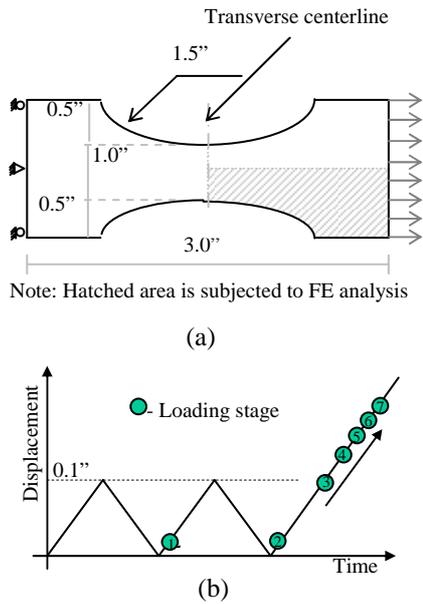


Figure 12: (a). Geometry of RBS specimen (b). Loading history (quasi-static)

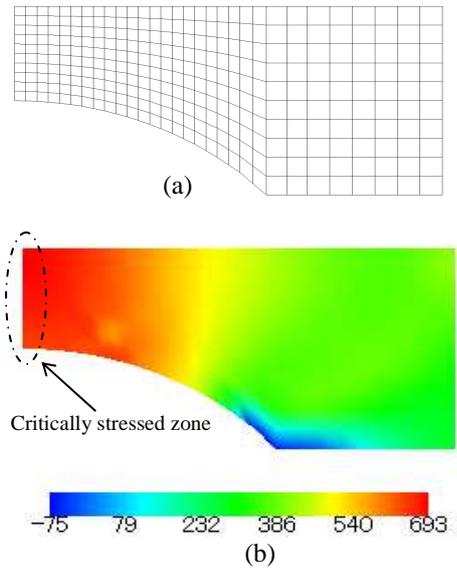


Figure 13: (a) FEM mesh (b) von Mises stress contours for monotonic loading

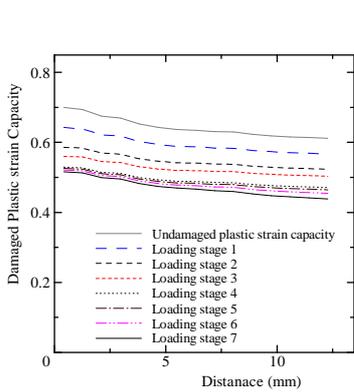


Figure 14: Damaged  $(\bar{\epsilon}_p^{cyclic})_{Critical}$  capacity

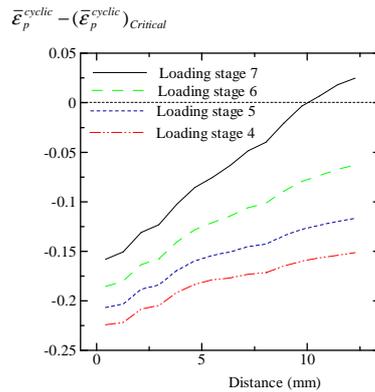


Figure 15:  $\bar{\epsilon}_p^{cyclic} - (\bar{\epsilon}_p^{cyclic})_{Critical}$  Variation

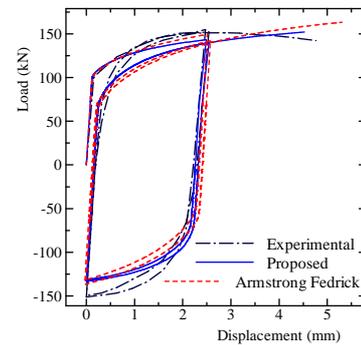


Figure 16: Load/displacement comparison

critical zone lies along the transverse centerline of the specimen as shown in Figure 13-(b). The considered material is, A572-grade 50 steel and toughness index ( $\alpha$ ) was taken as 1.18 [3]. Hence critical values of accumulated equivalent plastic strain ( $(\bar{\epsilon}_p^{monotonic})_{critical}$ ) at monotonic loading is calculated for sampling Gauss points along the transverse centerline of the specimen.

Table 2: Comparison of ELCF fracture displacement of RBS specimens

Description	Fracture displacement (mm)	Error with experimental displacement (%)
Experiment	4.76	-
ELCF Simplified criterion	4.50	5.46
ELCF CVGM criterion	4.72	0.84
Ductile fracture criterion	6.23	30.88

Then cyclic load FE analysis was conducted for one-fourth part of the specimen. The applied load (displacement) versus time variation is indicated in Figure 12-(b) and it has been predicted to simulate same effect of the experimental cyclic load test. Cyclically degraded values of the critical accumulated equivalent plastic strain,  $(\bar{\epsilon}_p^{cyclic})_{critical}$  variations are plotted at different loading stages as shown in Figure 14. Simultaneously the demands of accumulated equivalent plastic strain are also calculated for sampling Gauss points. Hence the  $\bar{\epsilon}_p^{cyclic} - (\bar{\epsilon}_p^{cyclic})_{critical}$  variations along the transverse centerline were determined and plotted as Figure 15.

Finally the prediction of ELCF macro crack initiation is made when  $\bar{\epsilon}_p^{cyclic} - (\bar{\epsilon}_p^{cyclic})_{critical}$  exceeds zero over the characteristic length ( $l^*$ ) at loading stage seven. The corresponding displacement to this instant is recorded as the fracture displacement, and the theoretical prediction of ELCF failure is compared with experimental results as shown in Table 2. The corresponding load displacement relations are drawn in Figure 16.

## 5. Conclusions

The general CVGM (cyclic void growth model) based extremely low cycle fatigue (ELCF) fracture criterion was simplified for steel structures where magnitude of the triaxiality remains relatively constant during their loading history. Then the ELCF fractures of few structural components were separately predicted by both simplified and CVGM fracture criterion and results were compared.

The comparisons of ELCF fracture initiation life of simplified model with previous CVGM model of some structural components reveal that simplified criterion produces reasonable accurate prediction to steel structures where magnitude of the triaxiality remains relatively constant. The case study exhibits that the simplified ELCF fracture criterion work well to obtain the more realistic predictions to ELCF fracture. Further, it determines the location of fracture accurately. Considering all these reasons, it is advisable to use simplified ELCF criterion to describe ultimate limit state of steel structures in seismic design practice. The main advantage behind this simplified criterion is that it can be easily utilized with commonly available elasto-plastic finite element (FE) packages. Because most of elasto-plastic FE programs produce outputs of simplified criterion dependent plastic variables such as accumulated equivalent plastic strain, stress components and effective stress. Finally, these reasons conclude that this study provide a new theoretical platform for structural design and maintenance communities by contributing convenient, precise and reliable ELCF criterion to describe real life of structures.

If indeed it is feasible to apply these ELCF proposed models at a larger scale, e.g. in large-scale beam column connections under cyclic loading. Large scale modeling of specimens often requires transition from smaller to larger elements to capture local (microstructure-level) as well as global stress effects. As a result, large-scale modeling, especially with cyclic loading, can be computationally very demanding. Advances in computational technology suggest that it would be worthwhile to explore the feasibility of applying such models at the larger scale. Even though some options are available, sub-

structuring method becomes as more convenient and such option based ELCF fracture predictions of large-scale structures are more appropriate for future studies.

### References

81. Kanvinde, A.M., and Deierlein, G.G., "Cyclic void growth model to assess ductile fracture initiation in structural steels due to ultra-low cycle fatigue", *Journal of Engineering Mechanics*, ASCE, Vol: 133, No: 6, 2007, pp.701-712.
82. Chi, W.M., Kanvinde, A.M., and Deierlein, G.G., "Prediction of ductile fracture in steel connection using SMCS criterion", *Journal of Structural Engineering*, ASCE, Vol:132, No: 2, 2006, pp.171-181.
83. Kanvinde, A.M., and Deierlein, G.G., "Void growth model and stress modified critical strain model to predict ductile fracture in structural steels", *Journal of Structural Engineering*, ASCE, Vol: 132, No: 12, 2006, pp.1907-1918.
84. Adasooriya, A.M.N.D., Siriwardane, S.A.S.C., Ohga, M., Kaita, T. and Dissanayake, P.B.R., "A simplified fracture initiation criterion for ultra-low cycle fatigue", *International conference on continuum mechanics, nanoscience and nanotechnology*, Peradeniya, Sri Lanka, 26-27 September 2008, pp. 87-94.
85. Siriwardane, S.C., Ohga, M., Dissanayake, R. and Taniwaki, K., "Comparative study for remaining fatigue life estimation of existing railway bridges", *Proceeding of 4<sup>th</sup> International Symposium on Steel Structures ISSS06*, Seoul, Korea, 16-18 November 2006, pp. 912-921.
86. Lemaitre J. and Chaboche J.L., "Mechanics of solid materials", UK, Cambridge University Press, 1990.
87. ABAQUS, Theory manual, version 6.5, Hibbitt, Karlsson, and Sorensen, Inc., Providence, R.I, 1998.

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