

**ANALYSIS AND INITIAL DESIGN OF CIVIL
ENGINEERING STRUCTURES USING STRUCTURAL
OPTIMIZATION**

Lowhikan S.S.

218079A

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Department of Civil Engineering

University of Moratuwa
Sri Lanka

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Declaration

I declare that this is my own work, and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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(Lowhikan S.S.)

The above candidate has carried out research for the Masters under my supervision.

Name of the supervisors: Prof. H.M.Y.C. Mallikarachchi
Dr. H.M.S.T. Herath

Signature of the supervisor: *UOM Verified Signature* Date: 20.03.2023

Signature of the supervisor: *UOM Verified Signature* Date: 20 03 2023

Abstract

Structural optimization of solids aims to find the optimal designs of structures by minimizing a constrained objective function such as the material compliance within a given problem domain. This constrained optimization problem is subjected to a set of displacement and load boundary conditions which in turn will be minimized with respect to a structural parameter. Although various structural optimization techniques have a sound mathematical basis, the practical constructability of optimal designs poses a great challenge in the manufacturing stage. The recent development in additive manufacturing partially side-steps this problem predominantly in the domain of Mechanical Engineering. However, in Civil Engineering structures, there is a great possibility of utilizing these optimization tools, especially in precast constructions. Currently, there is only a limited number of unified frameworks which output ready to manufacture parametric Computer-Aided Design (CAD) of the optimal designs. From a generative design perspective, it is essential to have a single platform that outputs a structurally optimized CAD model because CAD models are an integral part of most industrial product development and manufacturing stages.

This study focuses on developing a novel unified workflow handling topology, layout and size optimization in a single parametric platform (Rhino-Grasshopper) which outputs a ready-to-manufacture CAD model with the assessment of their structural integrity. In the proposed method, the first topology optimized pixel model is generated for any two-dimensional problem and converted into a one-pixel-wide chain model using skeletonization. From the obtained skeleton, a spatial frame structure is extracted, and then its member sizes and layout are optimized. Finally, the CAD model is generated using Constructive Solid Geometry (CSG) trees and its structural performance is assessed. In addition, industry-standard structural sections can be assigned to the CAD model to be analyzed and designed in accordance with standard codes of practice.

Keywords: Structural Optimization, Skeletonization, Spatial Frame Extraction, Computer-Aided Design Model, Structural Design

Dedication

To My Loving Parents:

Sivanantha Sarma

&

Komalathambigai

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S.S. Lowhikan

Department of Civil Engineering,

University of Moratuwa.

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