NONLINEAR ANALYSIS OF CABLE STRUCTURES

By



A THESIS

SUBMITTED TO THE FACULTY OF ENGINEERING
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE

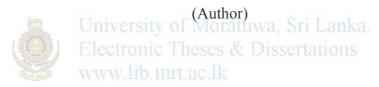
DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF MORATUWA
SRI LANKA

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DECLARATION

I hereby, declare, that the work included in this thesis in part or whole, has not been submitted for any other academic qualification at any institution.

H. C. T. Peiris



Certified by

Dr. I. R. A. Weerasekera
Supervisor/ Senior Lecturer
Division of Building & Structural Engineering
Department of Civil Engineering
University of Moratuwa
Sri Lanka

ABSTRACT

Large structures are widely used in the modern construction industry for infra-structure facilities development. Among these, long span structures with cables are becoming increasingly popular. In this category of structures deformations are large and estimations based on small deformation theory in the normal analysis are inadequate.

The large deformation analysis results in nonlinear behavior where principle of superposition does not hold. In geometrical nonlinear analysis, the equations of equilibrium are based on the deformed geometry after the load application. The length of a curved deflected line is longer than the initial length and the basic assumptions used in linear analysis may cause inaccuracies when the deformations are very large. It is also essential that bending effects of cables are considered.

Here we deal with large deformations, but small strain problems with linear stress-strain relationships. Although there are many methods found in literature to analyze cables exhibiting large deformation nonlinear behavior, it is hard to find a universal approach to describe the exact behavior of a cable considering all geometrical nonlinearity issues.

The analysis described in this study recognizes all such influences contributing to geometrical non-linearity. The procedure developed is versatile and gives a state-of-the-art analytical tool. This work fills a void in the current practice recognizing large deformation issues without any knowledge of small or large strains as opposed to what is required in commercial software.

A numerical solution procedure has been evolved to solve the resulting nonlinear non-homogeneous integral differential equation. The procedure is converging and a computer program has been developed for practical use. The results are compared with those in literature to validate the findings and to ensure the accuracy of the new large deformation nonlinear analysis technique.

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Finally I would like to acknowledge with fraternal love my colleagues and others who have assisted me in various ways whose contribution have led to the successful completion of this thesis.

H. C. T. Peiris

DEDICATION

To

my parents

and

all those who are interested and committed in advancement of science Electronic Theses & Dissertations

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ABBRIVIATIONS

Cross sectional area \boldsymbol{A} Matrix contains the derivatives of the displacements A_{θ} В Strain - displacement gradient relationship matirix / Equilibrium matrix CCarry over factor CC Compatibility conditions $[C_c]$ Compatibility matrix DNodal coordinate vector DDR Deformation displacement relationship $[D_f^1], [D_f^2]$ differential operator matrix \boldsymbol{E} Modulus of elasticity Material strain e, ε errors in finite difference Moratuwa, Sri Lanka. e_1, e_2 Equivalent modulus of elasticity E_{EO} external load vector fFIndependent member forces F_r Nodal forces due to temperature effects GL Green Lagrange Flexibility matrix of the whole structure $[G_{\mathfrak{s}}]$ $[G_t]$ geometry matrix Interval of finite difference technique hΙ Second Moment of area i,j,k,it,iii,ii,r integers ΙE Internal energy IFM Integrated Force Method

Jacobian transformation matrix

[J]

kUniformly distributed load along the member.(Catenary udl) Stiffness matrix [K], [S] $[\mathbf{K}G],[S_g]$ Geometrical Stiffness matrix $[\mathbf{K}_M], [S_e]$ Elastic/ Material Stiffness matrix l Horizontal distance between the supports LSpan Curved length of the member Original Undeformed length of the cable L_0 Undeformed length of the cable (in an iteration) $L_{\!\scriptscriptstyle U}$ MBending moment Number of divisions of the member in finite difference n N Axial force N_i / H_i Shape function sity of Moratuwa, Sri Lanka surface forces per unit area of the deformed body \overline{A} , p Force / Point load P p(x)Polynomial curve approximation for the profile equivalent nodal forces due to $\{q\}$ and $\{\overline{p}\}$. P_{eq} body forces per unit mass qUnbalance force/ Out of balance force QRadius of curvature Length of the member measured along the profile S S Rotational stiffness S, CSine and cosines $\overline{[S]}$ elastic stiffness matrix of the node $[\overline{S}_g]$ geometric stiffness matrix of the node $[\overline{S}_t]$ Tangential stiffness matrix of the node

Stdv Standard deviation TTensile force [t]Transformation matrix $T_1 T_2$ Force components along the axis of the cable TL**Total Lagrangian** Displacement within the element и V Volume Uniformly distributed load along a projected line of the member. WTotal load along the cable $\rightarrow X$ Coordinate axis for x measurements Measurement along X/ measurement in x direction of the of other points \boldsymbol{x} in finite difference technique when considered point is measured as X Xmeasurement in x direction of the considered point in finite difference techniqueniversity of Moratuwa, Sri Lanka. Coordinate axis for y measurements $\rightarrow Y$ Measurement along Y/ measurement in v direction of the of other points y in finite difference technique when considered point is measured as Y Y measurement in y direction of the considered point in finite difference technique $\rightarrow Z$ Coordinate axis for z measurements Measurement along Z Z. $\sqrt{T/FI}$ α Constant in finite difference β Constant in finite difference γ Small increment Δ δ Linear displacement of a point effective strain *3 Inclination of direction of load w to the normal axis of the cable η

θ	Angular deformation measurement
${\cal G}$	generalized internal deformations of the element
λ	cosines of the angle between the member local axis and the global axes
μ	Non dimensional stress parameter
ξ	Isoparametric coordinate
π	Total potential energy functional
ρ	mass density and
σ	Axial stress
υ	Tolerance value
ϕ	Inclination of direction of load k to the normal axis of the cable
φ	Inclination of the cable to the horizontal
Φ	Scalar function
$[\Omega]$	Compliance matrixy of Moratuwa, Sri Lanka.
ω	Angle of deviation at supports of a pinned supported member
(0)	(superscript) Value at the start of the current cycle
(1)	(superscript) Refers to the state at cycle end
0 in ,	(subscript, superscript) Initial states of the problem
e	(subscript) use to symbolize elemental properties
L 'NL	(subscript) Linear, Nonlinear component
[]	Brackets indicate rectangular or square matrix.
{}	Curly brackets indicate a vector.
1 11	First, Second derivative