

**FORM FINDING OF COMPRESSIVELY SELF-  
SUPPORTING SURFACES**

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Degree of Master of Science

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Sri Lanka

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Thesis submitted in partial fulfilment of the requirements for the degree  
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## **DECLARATION**

I declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any other University or institute of higher learning, and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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The above candidate has carried out research for the masters thesis under my supervision.

Name of the supervisor: Prof W.P.S. Dias

Signature of the supervisor:

Date:

## **ABSTRACT**

Compression-only surfaces are efficient structural forms as they fully utilize material at every cross-section. Thrust Network Analysis is a versatile tool which can be used for form exploration of compression-only surfaces. However, the form finding ability comes with the limitation of loading being in a single direction (e.g. gravity). As such, Thrust Network Analysis is not readily usable for form exploration under combined gravity and lateral loadings, such as seismic and wind loading. This research attempts to address this shortcoming by providing feedback to designers on the seismic capacity and wind loading capacity of the form while allowing them to carry out form exploration under gravity only loading condition.

For two dimensional arch forms and three-dimensional shells a procedure to provide real-time feedback on the thickness requirements of the form under seismic loading condition is proposed and implemented. For the case of a symmetric catenary arch subjected to a uniformly distributed loading a general solution for the thickness requirements can be given in a graphical format. It is further observed that the seismic loading capacity of compression only forms can be improved by bulking the supports.

A procedure is presented to provide similar feedback for two-dimensional compression-only surfaces subjected to combined gravity and wind loading. The feedback procedure and hence the usage of the feedback information in the design procedures differ for seismic and wind loadings, as seismic loads acts as a set of parallel loads and wind load acts perpendicular to the surface.

**Keywords:** compression-only forms, Thrust Network Analysis, quasi-static seismic loading, wind loading, design feedback

## **DEDICATION**

To the giants, upon whose shoulders I stand.

## **ACKNOWLEDGEMENT**

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