CHAPTER 3

Theoretical Developments

3.1 Principal of Sinusoidal PWM control

In sinusoidal PWM, the firing instants required to synthesize correctly the pulse width modulated wave are determined by comparing a triangular carrier wave and the reference modulation sine wave. The crossover points of the two waves determine the firing instants , as shown in the figure 3.1

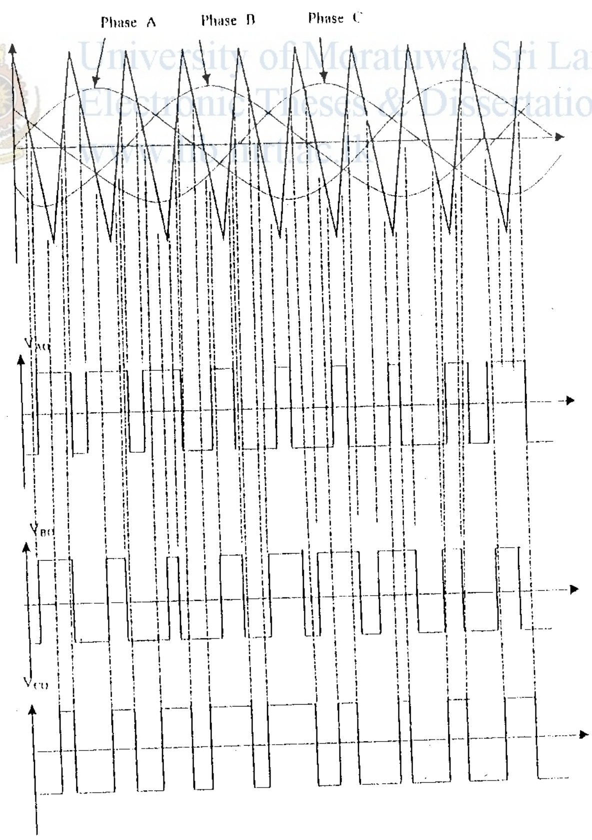

Figure 3.1- Generation of PWM wave

The pulse height of the pulse width modulated output signal is determined by the direct voltage on the supply side of the inverter. Also the pulse pattern is dependent on the ratio of the peak modulating voltage Vm, to the peak carrier voltage Vc. This ratio is called modulation index or modulation ratio, M

M = Vm / Vc =Modulation ratio

M is in its usual range 0< = M <=1

Another property of PWM waveform is the ratio between the frequencies of the carrier and modulating waveforms.

$$P = \frac{\text{Frequency of Carrier wave}}{\text{Frequency of Modulating Wave}} = \text{Carrier ratio}$$

According to these frequencies there are re two methods of sinusoidal PWM.

1. Synchronous operation

   If the frequency of the triangular carrier wave is an integer multiple of the frequency of the modulating sine wave, then the modulation is synchronous.

2. Asynchronous operation

   If the carrier frequency is not a multiple of the modulating waveform frequency, then the modulation is asynchronous.

   Numbers of switching pulses in each half cycle depend on the carrier ratio. Figure 3.1 shows how an increase in carrier ratio changes the number of pulses within each half cycle.

3.2 Inverter Model

The attention is drawn to 3- phase inverter since it is the commonly used inverter in the industry. The 3-phase inverter with its freewheeling diodes can be represented as shown in figure 3.2.

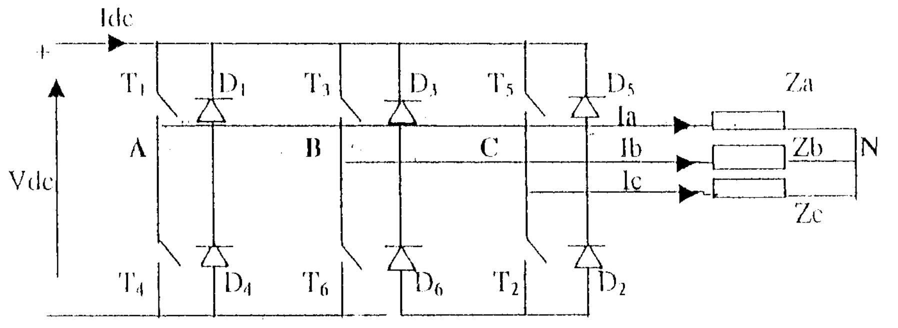

Figure 3.2 – Three phase inverter model with freewheeling diodes

T_1 to T_6 are the switching devices and D_1 to D_6 are the freewheeling diodes (used in order for inductive current to flow after particular switch is turned off) and Z_a , Z_b and Z_c represents the star connected load. The relevant phases are noted as A, B, and C and the neutral point as N.

The current flow is always taken towards the load and hence Ia , Ib and Ic take the direction as shown. For the analysis free wheeling diodes are neglected and the transistor switches are considered as ideal switches. Hence the simple model which we use is shown below;

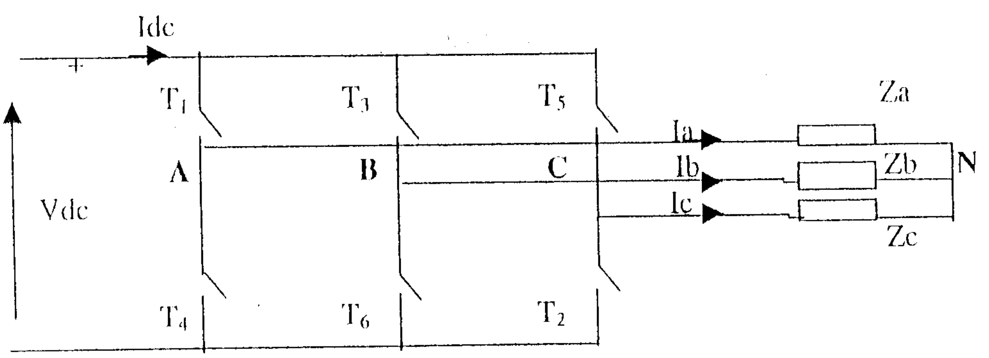

Figure 3.3 – Three phase inverter model without freewheeling diodes

The mid point of the voltage as "0" and hence the dc voltage is divided into two portions of Vdc / 2. The phase voltages are considered as Van, Vbn and Vcn.

3.3 Output Voltages

The output voltages are of three types; namely

1. Pole voltages : Vao , V bo , V co
2. Phase Voltages : Van , Vbn , Vca

These voltages can be obtained by analyzing the inverter model.

By applying Kirchof's voltage law;

Van = Vao + Von (1)

Vba = Vbo + Von (2)

Vcn = Vco + Von (3)

(1)+(2)+(3) ; Van + Vba + Vcn = Vao + Vbo + Vco + 3Von

= 0 (because the output is considered as balance)

hence Von = -(1/3) (Vao + Vbo + Vco) (4)

By using the 4^{th} equation in (1), (2) and (3);

Van = 2/3 Van -1/3 Vbo -1/3 Vco (5)

Vba =-1/3 Vao + 2/3 Vbo -1/3 Vco (6)

Van =-1/3 Vao-1/3 Vbo + 2/3 Vco (7)

This can be represented in matrix notation as;

$$\begin{pmatrix} Van \\ Vbn \\ Vcn \end{pmatrix} = 1/3 \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} Vao \\ Vbo \\ Vco \end{pmatrix} \qquad (8)$$

The pole voltages Vao,Vbo and Vco solely depend on the switching state and it can be written as;

$$Vao = \begin{cases} Vdc/2 \text{ when } T_1 \text{ is ON} \\ -Vdc/2 \text{ when } T_4 \text{ is ON} \end{cases} \qquad (9)$$

$$Vbo = \begin{cases} Vdc/2 \text{ when } T_3 \text{ is ON} \\ -Vdc/2 \text{ when } T_6 \text{ is ON} \end{cases} \qquad (10)$$

$$Vco = \begin{cases} Vdc/2 \text{ when } T_5 \text{ is ON} \\ -Vdc/2 \text{ when } T_2 \text{ is ON} \end{cases} \quad (11)$$

The line voltage as usually can be derived using phase voltages and hence can be represented mathematically as;

$Van = Van - Vbn$

$Vbc = Vbn - Vcn$

$Vca = Vcn - Van$

3.4 Micro Controller unit (MCU) Implementation

In the MCU implementation, the software is responsible for continuously updating the PWM duty cycles.

The PWM duty cycle calculations are based on a point in the sine wave. At full modulation (Maximum voltage), 100 percent duty cycle corresponds to the positive peak of the sine wave, and 0 percent duty cycle is equivalent to the negative peak. The zero crossover point is represented by 50 percent duty cycle, as can be observed in figure 3.4

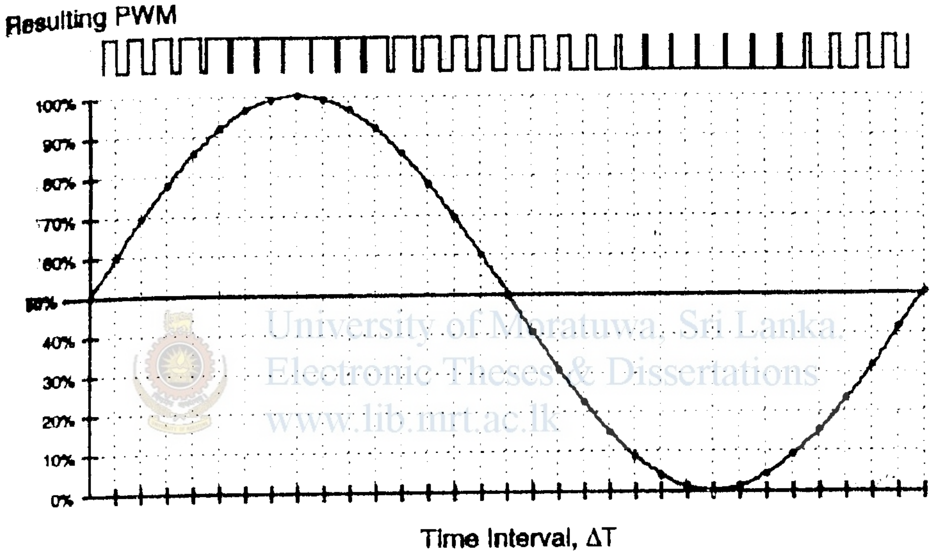

Figure 3.4 – Sinusoidal wave generation using PWM

A HIGH $= m. \sin(\Phi).50\% + 50\ \%$

B HIGH $= m. \sin(\Phi - \pi/3).50\% + 50\ \%$

C HIGH $= m. \sin(\Phi + \pi/3).50\% + 50\ \%$

Where m is the modulation index (0 to 1, 0= zero voltage, 1 = full voltage)

Since two PWM signals are required for each phase, the output duty cycle at the phase winding corresponds with the duty cycle of the top device. As an example, if the desired output at the phase winding is 75 percent duty cycle, the top PWM would be 75 percent duty cycle and the bottom PWM would be at 25 percent in actual practice, the PWM at the phase winding will be slightly different because of the necessity for dead time.

3.5 Changing output frequency

The PWM duty cycles are normally updated at a periodic rate, and the frequency of the sine wave determined by this up date rate (ΔT) and the number of samples points in a cycle. The relationship is given by

Output frequency = 1 /(update rate X number of samples)

To vary the frequency, either the number of sample points must be changed or the time between updates must be changed. When the number of samples is changed, the update rate is kept constant. The sample points typically come from a table, the number of sample points are easily changed by varying the increment value through the table as given by equation (3.1) The resulting frequency can be calculated as shown in Equation (3.2)

$$\textbf{Number of samples} = \frac{\textbf{Table size}}{\textbf{Increment value}} \qquad (3.1)$$

$$\textbf{Output frequency} = \frac{\textbf{Increment value}}{\textbf{Update rate X table size}} \qquad (3.2)$$

When the update rate is changed, number of samples is kept constant. The update rate is calculated by software, according to the speed set value of controller.