LEAKAGE INDUCTANCE CALCULATION
OF TOROIDAL TRANSFORMER USING
FINITE ELEMENT ANALYSIS

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Declaration

I declare that this is my own work and this dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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The above candidate has carried out research for the Masters under my supervision.

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Abstract

The researches done on calculating leakage inductance of a toroidal transformer were rare. In this research it is attempted to fill this gap by formulating a method to calculate leakage inductance of a toroidal transformer. Finite Element Method analysis is used to calculate the leakage inductance value for the horizontal plane of the toroidal transformer and estimate the leakage inductance of toroidal transformer. Open source magnetic finite element method software FEMM is used to implement the calculation model. The calculated values are compared against different winding methods used in toroidal transformer. Output of the model did show good correlation with normal toroidal transformer winding method which is being used in almost all the toroidal transformers with few exceptions of special winding methods. After introducing a correction factor it was able to achieve maximum error percentage of +/- 20% for the calculated values compared to the values measured from prototype samples. This is a good approximation when considering in a production batch of same transformer design, about +/- 10% variation is observed in measured leakage inductance values due production variations.

Keywords: Toroidal Transformer; Leakage Inductance; FEMM; finite element method
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List of Abbreviations

FEM   Finite Element Method
HV    High Voltage
LV    Low Voltage
I     Leakage Inductance
M     Mutual Inductance
L     Self Inductance
B     Magnetic Flux Density
Φ     Magnetic Flux
N₁    No. of turns in Primary Winding
N₂    No. of turns in Secondary Winding
R     Resistance
S     Reluctance
Lₘ    Magnetizing Inductance
V₁    Input Voltage
V₂    Output Voltage
I₁    Input Current
I₂    Output Current.