FUZZY APPROACH FOR SIZING PROJECT BUFFER; WATER SUPPLY PROJECTS IN SRI LANKA

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Degree of Master of Science

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Sri Lanka

May 2015

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Dissertation submitted in partial fulfillment of the requirements for the degree
Master of Science

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May 2015

DECLARATION OF THE CANDIDATE

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DEDICATION

Loving Yuhas,

Do a better work when you are grown up

ACKNOWLEDGEMENT

Along with the submission of this study I offer my sincere and foremost thanks to Dr Thanuja Ramachandran for guiding me towards the success of this endeavor.

Moreover, thanks are profusely disserved to the academic and non-academic staff serving the department of Building Economics, University of Moratuwa, for their selfless efforts in conducting a successful Masters Degree course in Project management.

My gratitude is always granted to Mrs C. J Siriwardena who always guides my carrier to the success in water supply projects. I further offer my thanks to Mr. S. Gunawardena, Mr. B.J katugampola, Mr R. Samaratunga, and Mr. J.M.D Gerad being the erudite luminaries who fostered thoughts in my initial survey to ascertain the eligibility of my study.

ABSTRACT

The intellectual views in the context of water distribution projects in Western province, Sri Lanka revealed safety time reservations are always hidden inside project durations imposed under project formalities. Task performers are unintentionally used to waste these safety reservations otherwise could have been exploited as time buffers in the light of uncertainties leading for delays. The study aims to forward an approach for sizing these safety reservations which are undercover and invariably intangible within project schedules. Nevertheless, the methods brought forward in literature under critical chain project management and associated schools of thoughts are found with common pitfalls as being arbitrary apportionments and having impotence for optimally resolving the resource contentions.

Water distribution projects within a predetermined range of magnitudes comprising an identified exhaustive set of recurring tasks are explored for the proposal. The proposed approach is limited for sizing project buffers in critical chains. By delving in to available techniques the study is resorted to fuzzy reasoning so as to avoid the prevailing backdrops under the existing methods of buffer sizing. The proposed approach shades lights on possibility and necessity functions according to the theory of possibility. Fuzzy sets are drawn for the task duration distributions built up of historical information. Fuzzy intersections are formed in between the defined fuzzy Guntt bars considering the precedence relationships between activities. The approach demonstrates the way for Alpha-cuts to drive fuzzy interval arithmetic operations to determine optimistic and pessimistic critical chain durations. Fuzzy sets are drawn to delineate the intersection between fuzzy resource availability (drawn considering the availability of number of task's specialized crews) and the fuzzy balance project duration (drawn considering the outstanding project duration along the timeline). Alpha level effecting on each fuzzy Guntt bar is determined minimizing the sum of violations to fuzzy subsethoods. These subsethoods are gained by fuzzy Guntt bars within the supersets formed by fuzzy intersections between resource availabilities and balance project duration. Minimizing the violations to fuzzy subsethoods is to be performed by an operational research technique known as non-linear programming. Once the sum of violations to subsethoods is minimized, the difference between the pessimistic and optimistic critical chain durations are accepted as optimized project buffer.

The proposed approach is empirically tested with a case study. The algorithm for; violations to fuzzy subsethood minimization problem, is performed with Microsoft Excel Solver Add-in tool. The proposed contention derives shorter buffer contributions for the tasks where fuzzy intersection between resource availability and balance project duration is having greater membership values and vice versa. The same scenario is optimally satisfied for all the tasks by minimizing the sum of violations to subsethoods. The approach resolve resource constrained project scheduling problem without shading lights on resource leveling. Inability of setting the project duration as a constraint in to the mechanism of non-linear programming problem debunks as an issue requiring exclusion. The new approach apart from giving recourse to a conventional fixed apportionment enables a desired orientation of optimization to be satisfied in sizing project buffer.

Key words; Critical chain project management, project buffers, theory of possibility, Alpha-cuts, fuzzy subsethood, non-linear programming

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LIST OF ABBREVIATIONS

- APRT Adaptive Buffer sizing Procedure with Resource Tightness
- ARBP Adaptive Risk Based Procedure
- CCPM Critical Chain Project Management
- CDF Continuous Density Functions
- CPM Critical Path Method
- C&PM Cut and Paste Method
- DI Ductile Iron
- FIS Fuzzy Inference Systems
- Nr Number
- PERT Programme Evaluation and Review Technique
- RCPSP Resource Constraint Project Scheduling Problem
- RD Relative Dispersion
- RF Resource utilization Factor
- RSEM Root Square Error Method
- SLRs Sri Lankan Rupees
- Sqrt Square Roots
- TOC Theory of Constraint
- TraNr Trapezoidal fuzzy Number
- TriNr Triangular fuzzy Number
- uPVC unplasticized Poly Vinyl Chloride

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CHAPTER 01

INTRODUCTION

1.1 Background

“Buffer” in its common assertion is an element associated with or ingrained in an entity, which foremost confronts with and copes against uncertainties and risks pernicious to the objectives of the entity. Buffer, at this confrontation is more likely being engulfed by the rival act, but urge to safeguard the mere objective of the principal entity. Thus, at the insignificant cost of the buffer, the core value is made immune from any vulnerability resultant by up fronting uncertainties. Oxford dictionary (2000) fostered thoughts for this preliminary illustration in view of “buffer”.

Whereas the buffer corresponds to a project schedule Howel, Hsiang, Liu and Russel (2012) explains “time buffer” as the time incorporated in to task durations so as to cope against uncertainty and to secure the entire schedule from diverse adversarial conditions.

Project programmes which are formulated in water distribution projects used to be the very rooted caliber providing measures to time management, reviewing progress, controlling resources, delay analysis and grasping viable alternatives for accelerations. They are fraught of recurring tasks associated with Pipe laying operations at various locations of the site i.e. alongside various public roads. Each recurring task is normally assigned to a specialized crew experienced in that specific trade. Timing and sequencing of these recurring tasks are at the discretion of the project manager. They are to be determined upon their activity dependencies and resources availability, since viable simultaneous operations are bountiful at any point of time within the project duration.

Time buffer, as an integral part of a project schedule in the context of water distribution projects in Sri Lanka is the mere substance of this study.

The most abundant custom in project planning and controlling in Sri Lankan Water Supply industry is that, schedules are created utilizing the entire contract duration allowed in the project formalities. Hence the task durations assigned to recurring tasks are invariably, as much as permissibly extensive within the project duration, being low risk high confidence estimates with greater pessimism. Project manager's awareness about the wideness of this ingrained safety time within recurring tasks i.e. activity buffer times, is indispensable at the same time he exercise project controlling. Conversance in terms of optimistic safety free activity durations appears to be a foremost concern apt to gain in practicing project scheduling in this sector of construction, unless pragmatic time estimates for recurring tasks would be remote from the project manager's circumspect. Jan and Ping (2006) corroborated, most of the duration estimates given for schedule activities are with hidden provisions for safety times to cope with any possible uncertainty.

Leach (2002) making a general emphasis which is as well attributable for the foregoing phenomenon in water distribution projects asserted; the common practice is distributing a concealed buffer among all the tasks instead of adding a schedule buffer at the end of a chain of tasks. This implies an empirical intuition is worthy of exerting, upon the buffer estimates which can optimally viz. most effectively, be sanctioned within the tasks. It further signifies the essence of assigning correct magnitude of buffers to the correct activities in project scheduling.

An approach for evaluating time buffers with higher credibility in water distribution projects necessitates a robust collection of statistics of projects meeting diverse project related variable conditions. Long and Ohsato (2007) denoted; uniqueness and lack of historical data are likely to hinder, assessing the degree of uncertainty of task durations based on probabilistic methods. This reveals, collection of statistics to plot a distribution function of each recurring task's duration estimates is at the very threshold of establishing a buffer sizing approach.

An approach for effective buffer sizing, positioning and managing thus necessitated in water distribution projects, such necessity yields for having recourse to theories conceptualized in the past. Therefore this study rummages in to the ideologies built in Critical Chain Project Management (CCPM) theories.

CCPM ushered in Goldratt (as cited in Goto, Takahashi, Takei, and Truc, 2012) conceptualize the principles of buffer sizing, buffer management and resource conflicts in view of project planning. Advent of CCPM is said to have drawn subsequent research interests towards such disciplines. CCPM submits a method of adding time buffers in to project networks in order to absorb the impacts of uncertainties to secure the promised due date.

Leach (1999) proclaimed CCPM; "it reengineers project planning and management to eliminate common problems" (p.39). It further encapsulates that, CCPM focusing on project schedules deter the variations to task sequencing, cost overruns, uncertainties of activity durations and resources conflicts.

Eksioglu, Rom, and Tukel (2005) stressed that, buffers are estimates for the extent of uncertainty associated with task durations. Francis (1999) throwing more lights on CCPM concepts rendered that, schedules shall be equipped in order to cope against unnecessarily wasting the safety time embedded in task durations. This is said to be possibly done by extracting such task related safety time, leaving the task duration an aggressive estimate and then to accommodate the accumulated safety at the end of the critical chain as a time buffer.

Jan and Ping (2006) stressed that, Goldratt having founded CCPM fundamentals presents "Critical Chain", a novel concept to the Project Management. "Critical Chain in a project is defined as the longest sequence of activities from several different paths, connected by activities performed by common resources" (Jan & Ping, 2006, p.858). "The Critical Chain is the set of tasks which determines overall project duration, taking in to account both precedence and resource dependencies" (Eksioglu et al., 2006, p.401).

"CCPM protects the overall project delivery time with a project buffer at the end of Critical Chain. This exploits the statistical law of aggregation by protecting the project from common cause uncertainty of the individual activities in an activity path using buffers at the end of the path" (Leach, 1999, p.46).

Apart from accounting and assigning buffer for each individual recurring task, the former studies by Francis (1999) and Leach (1999) conforming to CCPM theories advocate, adding an accumulated buffer by integrating each individual recurring task's embedded safety time to the end of the Critical Chain. Hence identifying the Critical Chain occupied in a project schedule has attracted as much as the same importance as sizing and positioning time buffer according to CCPM theories.

Leach (1999) states "CCPM project planning and control process directly addresses uncertainty and variation in project activity duration" (p.39). Attesting the same Howell, Hsiang, Liu, and Russell (2012) mention "We define "time buffer" as time added to task durations to compensate for uncertainty and protect against variation"(para. 1).

It is therefore explicit; the concept of "buffer" within CCPM principles an estimate of the time duration which is complementary for uncertainties and variations inherent in the activity durations. This forms the basis to investigate; based on their level of uncertainty and biasness for variation, the time available in each recurring task's activity duration that can contribute for the buffer at the end of Critical Chain.

"Goldratt suggests a very simple method to size buffers; use one half of the sum of activity durations in the chain of activities that precedes the buffer" (Leach, 1999, p.46). Long and Ohsato (2007) having reference to the Goldratt in his school of CCPM stated the magnitude of buffer at the end of the Critical Chain as 50% of its length.

This was subjected to the contention; "Yet the size of these buffers is usually specified as an arbitrary fraction of the estimated chain duration" (Trietsch, 2004, p.267) and "Goldratt suggests the size of buffer is 50-50 estimated duration. However, 50% estimation is too arbitrary and difficult to apply in construction real world" (Jan & Ping, 2006, p.861).

These counter justifications envisage; 50% estimate of activity time durations as the buffer contribution is not reasonably credible in terms of water distribution projects under the current study. A converse argument can further be raised thinking contrary to the CCPM theory; how a one can ensure that, there is no buffer incorporated within an activity of having its duration at 49% confidence as per its duration distribution. Nevertheless, 50% contribution from each activity duration is suspicious being reasonably sound for all the scenarios, to become commensurate in integrity for an optimized project buffer. Because 50% duration estimates in its essence do not uphold for buffer optimization.

Prevailing highly stochastic and arbitrary procedure of the buffer sizing under CCPM principles which is furthermore prejudicing to be optimal is an identified problem, resolution for which is extending to the very root of the current study.

In another aspect of the problem, a buffer sizing approach shall ensure that, once the safety time is removed, the outstanding safety free time durations for the activities to be realistic. "The safety time for each activity is the protective estimation minus reasonable estimation" (Jan & Ping, 2006, p.861). Steyn (2000) highlighting the importance of safety free activity time to be realistic, elaborated the phrase "safety free activity duration with buffer", as the same in meaning with "Just In Time (JIT) with buffers" (p.367). He further asserted under CCPM concept, contingency reserves in individual activities are exploited in a way the outstanding activity durations would remain as aggressive but realistic estimates.

Eksioglu et al. (2005) exposing another shortfall in CCPM emphasizes, CCPM does not submit a specific procedure to unravel the resource conflicts in activity paths. Long and Ohsato (2007) addressed the resource constrained project scheduling problem via a buffer sizing and re-sequencing method as a mean of streamlining the Critical Chain, buffer under their study is sized and positioned according to the resource constraints and the uncertainties inherent in the activities.

Hence, according to the both presiding paragraphs "soundness of the safety free activity durations" and "resource constraints within activities" are conceived as such priority problems to be resolved by further researches developed under the subject of "buffer with CCPM theories".

Neither CCPM principles nor buffer sizing methods are known to be practicing in water distribution projects. Buffer on the other hand would most convincingly be a lurking entity ingrained in deep clandestine within the project programmes. Hence what has been determined as affordable under the current research is "making a divulgence to the optimal buffer aggregation, accounting from each activity's individual buffer contribution integrated along with the principles of CCPM".

1.2 Research problem in brief

Water distribution projects in Sri Lanka are in need of a project buffer sizing approach. Nevertheless, the existing buffer sizing approaches are of identified lapses resulting in; solutions with arbitrary apportionments, unreasonably aggressive task durations, schedules incompatible with resource constraints and poor rationalities in computation. Therefore, the problem before the current study is the absence of a suitable buffer sizing approach for water distribution projects free from the aforesaid inefficiencies.

1.3 Aim of the study

- To submit an approach for estimating the size of optimal project buffer allowable in water distribution projects in Sri Lanka

1.4 Objectives of the study

- Identifying the presence and the requirement of sizing project buffers in the context of water distribution projects in Sri Lanka
- Identifying common inefficiencies by critically analyzing the existing buffer sizing methods set out in CCPM ideology and other associated works
- Setting up a criteria to assess the variability of durations possessed by recurring tasks within their appropriate resource contentions inside the project duration in relation to water distribution projects
- To propose an optimization technique in determining best time apportionment between the pessimistic and safety free optimistic recurring task durations under resource and project duration constraints in water distribution projects

- To propose an reasoning method to replace existing 50% or such other arbitrary fractioning so as to ascertain the optimum buffer reservation in task duration estimates within the phenomenon of water distribution projects

1.5 Scope and limitations of the study

- The study is limited to the Water Distribution projects implemented in Sri Lanka compatible with the underlying profile
  - Overall project durations less than three years
  - Nominal line diameters varying between 60mm and 400mm
  - Value of measured works excluding all taxes, contingencies, provisional sums and preliminaries from contract sums and excluding the supplying cost of material to be worth less than Sri Lankan Rupees (SL.Rs) 900,000,000.00

1.6 Research Methodology

- A literature survey is conducted to identify the existing approaches in buffer sizing
- Critiques are drawn to the apparent deficiencies in the existing approaches and their inertia for success
- An afresh buffer sizing approach is proposed to exploit the existing deficiencies by having recourse to a justified technique of fuzzy reasoning combined with an operational research technique to arrive at a optimized solution
- The proposed approach is verified for its practicality by exercising a case study with a set of water distribution projects chosen inside the range of limitations to the study

CHAPTER 02

LITERATURE REVIEW

2.1 Introduction

The chapter discusses the concepts relating to CCPM. It is critically reviewed the existing ideologies presented in literature and their eligibility is verified for applying under the current research. It is identified the lapses prevailing in CCPM in terms of buffer sizing in order to codify the research problem.

2.2 Inbuilt theories of CCPM

Leach (1999) explicated three theory tools which formed CCPM rationale. These theories are explained as drawn to provide guidance to eliminate identified six project specific undesired effects which lead the project schedules to over run. These three theories are,

a) Theory of Constraints (TOC)
b) Common cause variations
c) Statistical law of aggregation and central limit theorem

2.2.1 Theory of Constraints (TOC)

Goldratt (as cited in Leach, 1999) summarized TOC as "any system must have a constraint, otherwise its output would increase without bound, or go to zero" (p.40).

"In his 1984 book titled *The Goal,* Eli Goldratt began advocating the concept of the Theory of Constraint (TOC) for managing manufacturing operations. The theory acknowledges that every system has a constraint and that, to increase the system performance, performance at the constraint must be improved. Attempting improvement elsewhere is thought wasteful because the constraint will remain the bottleneck" (Kokoskie, 2001, p.2).

Preceding statement pronounces, to transform a particular system's performances better in contrast to the expected results, it is the system constraint required to be addressed and made efficient. This prime theory summons the empirical intuitions to orient towards the identified system constraint which leads for project time overrun to be exploited in project planning.

2.2.2 Common Cause Variation

Deming (as cited in Leach, 1999) identified two causes of variations which possibly take effect in project scheduling, which causes may result for deviations in project implementation from its plan.

a) Common cause variation; a cause inherent in the system
b) Special cause variation; a cause specific for a particular resource or group of resources assigned to a project

Leach (1999) highlighting that common cause variation as the only pertaining cause of variation for the contention developed in CCPM noted, common cause variation represents the uncertainty in activity performance time but not exceptional as discrete project risk events.

This renders CCPM takes only in to account the statistical variances inherent in activity duration estimates. Such deviations which can possibly take effect due to project or activity specific characteristics are away from CCPM ideology.

2.2.3 Statistical law of aggregation and central limit theorem

According to Leach (1999) statistical law of aggregation is attributable to the principles of CCPM as; variance in project duration is the same as the sum of individual activity variances along the activity path forming project duration.

“If a number of independent probability distributions are summated, the variance of the sum equals the sum of variances of the individual distributions” (Steyn, 2000, p.365).

Kokoskie (2001) unveils if the variance calculated from an activity duration distribution is greater; it is more likely to expect a longer deviation from its real duration in contrast to its mean estimate. This extends to the breath that activities of greater variances in their durations are comparatively poor in deriving accurate duration estimates.

Moore and McCabe (as cited in Leach, 1999) defining central limit theorem stated, sample mean coincides more with the mean of a normal distribution as much as its size increases, this implies together with the law of aggregation that, activity path forming the project duration is more susceptible towards a symmetrical distribution.

Leach (1999) exemplified; adding an aggregated one standard deviation from each activity duration estimate in to this 50% confidence project duration estimate would derive a sophisticated buffer for the project duration. This attempt is further demonstrated as being capable in curtailing the entire project duration from its initial low risk high confidence estimate.

Steyn (2000) drawn in conclusion that logic reasoning created in CCPM leads for reduced project duration.

"As a result of the effect of aggregation this buffer is smaller than the sum of the individual reserves that have been removed from the low level activities. Thus project duration is reduced. The higher the number of activities on the critical path of a project, the more project duration can be reduced" (Steyn, 2000, p.365).

2.3 Undesired project effects for exclusion

Goldratt (as cited in Leach, 1999) identifies the core problem which leads for project failures as "failure to effectively manage uncertainty", leading for six undesired effects to overrun the project schedules.

Taking above facts together in to account, three theories fraught in CCPM are to "eliminate six specific undesired project effects" and the fact that, "failure to manage uncertainty leads for these undesired effects", it divulges that CCPM three theories are seeking resolutions for uncertainty, particularly in relation to activity duration estimates based upon the influences of common cause variations.

2.3.1 Excessive activity duration estimates

"Most project managers include contingency time within each activity estimate to account for individual activity common cause variation. Contingency is defined as the difference between 95% probable estimate and the 50% probable estimate" (Leach, 1999, p.42).

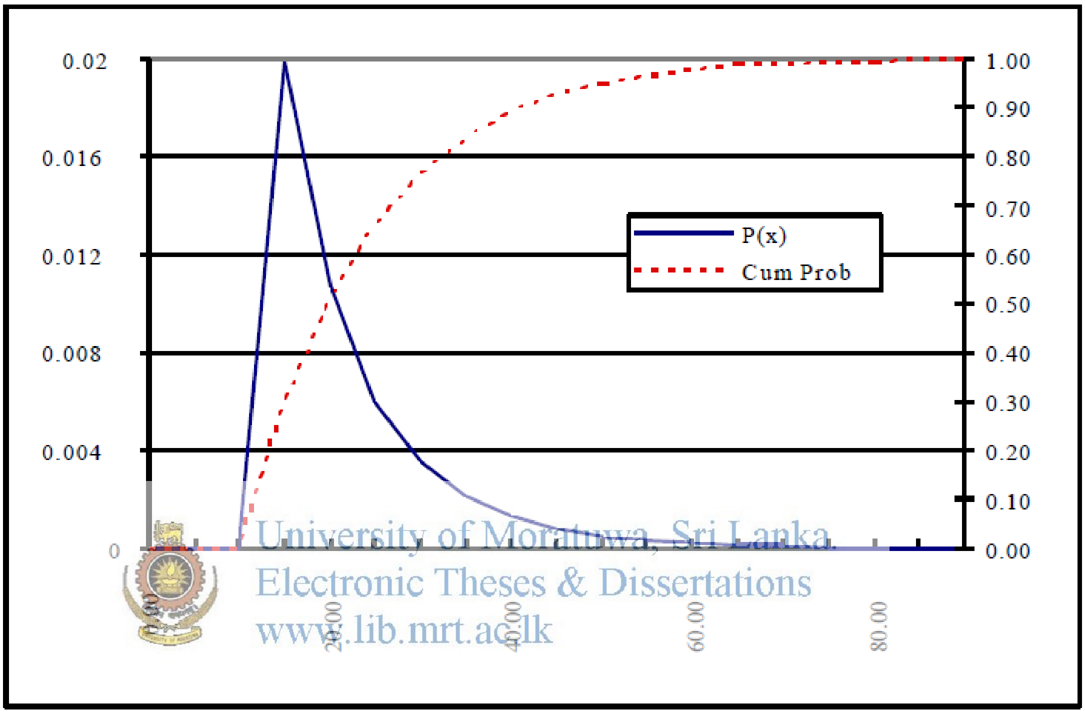


Figure 2.1: Typical project activity time probability distributions
(Source: Leach, 1999)

Figure 2.1 depicts a typical probability distribution of an activity duration estimate. According to the elaboration driven by Leach (1999) upon this figure, leftward skew with long tail to the right presents the distinct features of the common cause variation; hence the law-risk estimate could be several times lengthier than the 50% probable estimate.

Rand (2000) reasoning out existence of a safety time within activity durations urges, "If safety time is built into each estimate, this would imply that, when all activities are linked together, there should be a high probability of completing on time"(p.175).

Eksioglu et al. (2005) affirmed added safety at each activity durations are likely to waste, reflecting local efficiencies to inflate the project make-span i.e. global goal of the project. This implies safety time added to a project shall have to be oriented towards its entire objective than utilizing to adopt the efficiencies in individual activities.

2.3.2 Student syndrome (Parkinson's Law)

Leach (1999) mentioned, if activity durations are 50% probable, 50% of the activities shall have completed early than the planned completion date. But in contrary, most of the activities are usually completed on the schedule meanwhile another significant portion of them are reported as late completed.

Merdith and Mantel (as cited in Leach, 1999) stated drawing their contention to collate with Parkinson's law, activity time performances are used to expand filling the additional time available and wasting the safety time reserved for the activity. Goldratt (as cited in Leach, 1999) ushered in this human behavior as "student syndrome".

Foreseeing the possible aftermaths of adding excessive contingency reserves in to activities Steyn (2000) stressed, work is likely to expand filling the excessive redundant time available, at this juncture quality would be the more concerning fact than its actual requirement.

2.3.3 Failure to pass on positive variations

"Projects do not get the benefit of many actual early activity completions. Even if completed early, performing resources often fail to pass on positive variations" (Leach, 1999, p.42).

Steyn (2000) presented the underlying three reasons for the failure to handover any positive variations in activity early completions to the succeeding activities.

a) Unsolicited time spent for reviewing the works after early completion, instead of reporting early completion
b) Unavailability of all the resources for succeeding activities at the time predecessor activities finish early

c) Need to secure the credibility of the extensive activity duration prior negotiating with and sanctioned by the management

2.3.4 Activity path merging

It is demonstrated by Leach (1999) that, projects have multiple activity paths converging in to the critical path, abundantly near the milestone denoting the project completion due to certain operations (like testing and commissioning) which invariably prolong until the project end. Successive activities which are waiting for the completion of all presiding activities are argued accompanying the longest delay rather than any of the positive variations from merging paths feeding in to the critical path.

Critical path as denoted by Leach (1999) is; the longest chain of activities in a project plan without any slack time. Critical path is further mentioned to be susceptible for varying, whereas the activities constituted in other paths which merge with the critical path being delayed and subsequently protracted.

2.3.5 Multitasking

"Multitasking is performance of multiple project activities at the same time" (Leach, 1999, p.44). Multitasking is moreover disdained as a practice leading for protracting time for completion in view of every individual activity within the multitask group. Multitasking is claimed to be a mere attempt of imparting time in between the activities, when completed multiple tasks are submitted, the time consumption would be the same as the summation of time consumed by each individual activity. This is distinguished from simultaneous multiple tasks done by separate groups of resources in parallel activities in the project plan, whereof in multitasking they all are performed by a single group of resources. When multitasking merges with the critical path, an overall project delay is supposed.

Kokoskie (2001) advocates elimination of multitasking as a way of expediting the inflow of revenue. Because each activity in the group of multiple tasks gets completed before the sum of durations of all the activities in the group i.e. entire duration of multitask. Therefore revenue pertinent for each sub task is earned in interim payments prior the time that multitasking gains the same amount in aggregation at the end of accumulated time of each activity. Therefore elimination of multitasking reflects a healthy cash flow.

It comprehends that conveyance from pessimism to optimism results in wasting the contingency reserves via the statement "when contingency reserve is built in to the schedule for an activity, the pessimism that prevails during the planning stage seems to be replaced by optimism during the period following the start date" (Steyn, 2000, p.366). This is further explicated as it is the impression of pessimism which compels to build in contingency reserves within activity durations, but optimism which comes in to existence wastes this contingency reserve at the outset of the activity performance stage by working on other critical external tasks. Eliminating multitasking is mentioned thereof as an exercise which drive focus on the system constraint i.e. project critical chain, which enables subordinating non-constraints to exploit the constraint.

2.3.6 Loss of focus

Leach (1999) contended, project managers are ignorant of where to focus on their project plan and argued,

a) Early start of all activity paths including non-critical paths unnecessarily early for project completion could waste embedded slack time i.e. float time
b) Changing the critical path during the project performance misdirect the focus.
c) Focus on "earn value" as a tool of progress evaluation only indicates the monitory value of the progress, but it losses the concentration on difference between critical and non-critical paths

The above study denoted, elimination of early start times of the activities enable the planning team to determine the activities which need to be given higher priority for constrained resource allocation. This exercise excludes to some extent the potential for simultaneousness between activities particularly at their outset. But, any act of mitigation via managing available slack time seems abstained.

2.4 CCPM planning process

Planning process suggested under the CCPM principles are based on five focusing steps centering the system constraint, as outlined by Leach (1999) they appear as,

i. Identify the system constraint
ii. Exploit the system constraint
iii. Subordinate everything else to the constraint
iv. Elevate the system constraint
v. Repeat the process in the presence of a new constraint

In view of the above procedure presented under CCPM, the system constraint is the prime issue to be resolved by the project plan. Therefore the correct identification and implementation of foregoing sequential procedure in addressing the system constraint is the suggested method of planning in CCPM.

2.4.1 Identify the System Constraint

Goldratt (as cited in Leach, 1999) identifies the critical chain as the constraint of a project. Sequence of the longest dependant events determined by activity dependencies and as well resource dependencies which prevent the project being completed in a shorter period is earmarked as the critical chain i.e. the project system constraint. Thus addressing the system constraint is pronounced as a resolution for resource constraint, where in the absence of a resource constraint, project critical path is said to be the same as critical chain or the system constraint.

"The chain concept acknowledges that a particularly scarce resource could make the task not on the critical path become part of the critical chain" (Kokoskie, 2001, p.3). Critical path is thus distinguished from critical chain which is furnished with resource logic under CCPM. Barnes, Dvir, and Raz, (2003) defined "critical chain" as the resources leveled critical path and "critical resources" as those resources required by the critical chain tasks. Globerson (as cited in Barnes et al., 2003) noted once the constraints exerted by the resources are concerned critical path deviates from its original formation and thus transforms to critical chain. A resource leveling algorithm has to be exerted according to the view presented in Barnes et al. (2003) in order to identify the critical chain or the system constraint. CCPM does not exert insight in to any resource leveling algorithmic procedure. Nevertheless, resource leveling algorithm which is invariably a heuristic approach requiring an optimum solution is dictated as a matter in need of a solution.

Reconciliation has to be drawn between the schedule (time), scope and cost (resources) when outlining the system constraint. Leach (1999) pointed out when schedule increases with fixed deliverable scope, cost increases. As the scope increases with fixed cost, schedule increases. As the scope increases with the fixed schedule, cost increases.

Nevertheless, retaining cost at a constant whilst increasing both scope and schedule seems contentious and out of empirical certainty. Elmaghraby, Herroelen and Leus (2002) stating the same fact in a different dimension notes "reduction in duration of an activity, when feasible, usually comes at a cost" (p.302). A time-cost trade off problem is given a value under the later study in the light of an attempt by the project management to reduce the project duration.

2.4.2 Exploit the system constraint

Leach (1999) elevates the procedure suggested in CCPM sequencing of which is outlined below. The procedure exploits the statistical law of aggregation to determine and positioning a project buffer at the end of the critical chain in order to safeguard the system constraint from common cause variation.

a) Assemble the plan with low risk activity duration estimates calculated from the project resources
b) Reduce each activity duration at its average i.e. 50% probable estimate, meanwhile its activity dependencies exist as the same
c) Activity dependencies shall have ensured at this stage that, the way they are connected is devoid of multitasking, and have all the resources available when started

d) Buffer is computed and positioned at the end of the activity path as the aggregated difference between the low risk and average activity duration estimates

Barnes et al. (2003) exposes the foremost step in CCPM project planning as creating a list of tasks constituent of their duration estimates and dependencies in between. Forming dependencies, task precedence relations and resource availability are the facts drawn for focus. For the latter step resources leveling is advocated.

Concentrating on the final step of the foregoing sequence, Steyn (2000) asserts, emphasize shall have to be more towards the positioning of the contingency reserve rather than its redundancy. The study further held that preference to be given to the project over the individual activities in positioning the time contingency reserve.

Traditional project scheduling techniques i.e. Programme Evaluation and Review Technique (PERT) and Critical Path Method (CPM), according to Goldratt (as cited in Kokoskie, 2001) are irrational due to their focus centering to local optimization instead of system optimization and safety time being allowed for all activities instead of devoting for the entire project as a whole. Neither these traditional project scheduling techniques are adopted for resource constrained project scheduling problem as indicated by Elmaghraby et al. (2002). This unaddressed problem presents in traditional project scheduling techniques seems to have induced CCPM to seek for different planning techniques addressing resource constraint.

Goldratt (as cited in Eksioglu et al., 2005) claims the bottleneck of a project as the critical chain which shall be protected by adding buffers. “The aggregated project buffer inserted at the end of the critical chain provides for contingencies on all critical activities” (Steyn, 2000, p.366).

“As mentioned before, buffers can be added to a project as artificial activities and thus they change the structure of a project. Furthermore, the methods suggested in the literature do not incorporate project characteristics when determining buffer sizes” (Eksioglu et al., 2005, p.403). There it is made evident that, a buffer is a certain item which is not possessed by a specific activity in the work breakdown structure forming the project programme, but acquires time similarly as other activities do, insertion of a buffer changes the project structure from timing in terms of activity durations, starting times, resource allocation and slack time but not in relation to activity dependency. Buffers are thus added to revamp for common cause variations, this is corroborated by the fact; their sizes are independent from the project related characteristics.

Elmaghraby et al. (2002) denoted shortening of activity duration estimates to its 50% confidence level would leave single point activity duration at its median estimate. As in fact “50% confidence duration estimate” being the central focus, “median duration” appears more appropriate to its essence than “average or mean duration” referred in Leach (1999).

Kokoskie (2000) explaines setting out activity durations at 50% confidence estimate would enable half of the activities to under-run and the other half to over-run the planed activity duration. A substantial justification is mooted from there which attests reduction of project duration is viable with CCPM ideology, a shorter duration would be reasonably sufficient to accommodate the balance half of the probable activities which over-run the activity durations. Advocating this Barnes et al. (2003) noted “statistical independence” which explains that, at 50% confidence level of duration estimate 50% of the tasks under-run whilst the balance 50% of the tasks overrun. It further affirmed the overall reduction in total project duration is tangible. This is substantiated stating that, the standard deviation of an accumulated group of independent random variables i.e. in this context the project duration formed with individual activity durations, being less than the sum of standard deviations of each individual variable. Conforming to law of aggregation this renders; overall project duration formulated with 50% confidence duration contribution from each activity summed up with aggregated standard deviation i.e. project buffer in this context, leads to a reduction in project duration in contrast to its initial high confidence estimate.

Nevertheless, at this juncture literature presents substantiations also in counter for the difference between safe and average duration estimates, as the buffer size presented in CCPM. Eksioglu et al. (2005) doubt 50% of the safe time estimate reasonably corresponds with the variability in activity duration estimates. Right long skewed nature of activity duration distributions which accompanies the perception that, activities are liable for lengthier durations than 50% duration estimates, is presented as a supportive clue for this argument. On the other hand, significant frequency of activity durations is stated in that examination as likely falling short behind 50% confidence duration estimate. Barnes et al. (2003) in another examination criticize reduction in all activities in a fixed percentage as suggested in CCPM. It is claimed, that people do not customarily overestimate the durations of different activities with the same proportion as CCPM assumes.

Liu, Tsai and Wei (2000) urged 50% reduction in chain duration would violate the initial schedule portrayed by the project manager. It is said as prone to vary the resource schedule initially demarcated, to cure the problem the study suggests buffer sizing based on existing resource constraint within the schedule.

Hence to commence with CCPM approach the foremost exercise to do is to establish the most envisaged project activity dependencies. CCPM does not seem to have extended its concepts to advice in developing rationale links in between the activities. Once these dependencies are developed they would be intact throughout the subsequent CCPM steps. Nevertheless, CCPM emphasizes the need for assurance that there is no multitasking or resource scarcity according to the resource schedule. CCPM established a 50% arbitrary confidence estimate i.e. median estimate, as the planed duration for each activity and brought the different between low risk estimates computed for the initially drawn plan and this median estimate as the time contribution for the project buffer from each activity. CCPM here does not forward reasons for establishing 50% as the confidence estimate for the activity durations free from all ingrained safety, other than 50% level of confidence being an average estimate based on statistical reasoning. Therefore a counter argument which questions "whether 49% level of a confidence does really incorporate any inherent safety within the activity duration" would obviously gain an absolute value for concentrating.

2.4.3 Subordinating to the constraint

Though "early start" of project activities gear exclusions for project risks Leach (1999) stressed that, CCPM advocates for "late start" of activities. Late start is thereof highlighted as initializing for;

(a) Alleviating the negative implications possibly occasioned by the changes effected in already performed activities
(b) Securing the slack time possibly being wasted due to early start of the activities
(c) Eliminating simultaneous implementation of multiple activities which relegate the extent of priority necessarily to be attributable for the project critical path

The study forming the above illustration then accompanied CCPM rationale to the point where, it recommends equipping the activity feeding paths which connect to project critical chain with late start of feeding activities and a "feeding buffer" to cope against their respective time variances. Feeding buffer broaden thereof as a maneuver of immunizing critical chain from delays occur in feeding paths. Thus it is explained the recognized principle in CCPM; critical path does not change over the project performance.

"CCPM concept advocates that the critical chain does not change once the project is started" (Kokoskie, 2001, p.2).

Steyn (2000) noted that CCPM formulates all non-critical activities to be planed as late as possible with feeding buffers in order to subordinate non-constraints to exploit the constraint. There it is underlined; exploitation of the system constraint is made immune from delays that occur in non-critical path by positioning a feeding buffer where non-critical path adjoins the critical chain, making the critical chain a flow of dependant events which never varies. It is further drawn the fact in conclusion; CCPM which converged attention towards inertia to change, generates schedules which are rigorous during the execution.

Barnes et al. (2003) illustrated that, feeding buffer being the aggregation of safety time inherent of the activities in the feeding chain. Slack time at this explanation is said to be eliminated by the late start of the activities.

CCPM introduced "resource buffer" to exploit the resource constraint to safeguard critical chain from its vulnerability, contemporizing for this Leach (1999) underlined that, resource buffers alarm to ensure resources are readily available to deploy in critical chain activities. Subsequently with the input dependencies affiliating from presiding activities are accomplished critical resources are said as required to be duly available to be deployed in critical chain activities. Resource buffers moreover denominated as resources flags are only to be housed in critical chain as indicators devoid of any time implication.

Sakar (2012) drawing cognizance to resource buffer urges, "these buffers are positioned in places wherever a particular resource has a job on the critical chain, and the previous critical chain activity is done by a different resource" (p.5).

Steyn (2000) widened the term resource buffer or resource flag as a mean of advanced warning to acknowledge the impending time on which inputs necessary for the succeeding activity being ready as the presiding activity finishes. "The resource buffer does not actually consume any resource, and it adds neither time nor cost to the project" (Barnes et al., 2003, p.28). Albeit, the presiding study argues resource buffer, in the making of its rational had been unanswered to certain probable confrontations. How to handle the resources under outside contractor's custody and to manage unscheduled communication in exploiting resource buffer are the unanswered matters being posed by the foregone study. Liu et al. (2000) relegates the resource buffer proposed in CCPM concept, due to the claimed fact that it is devoid of any established guidelines, how to institute in to the project schedules.

Bevilacqua, Ciarapica, and Giacchetta (2008) enlighten that, resource buffer provides protective capacity in to schedules by ensuring critical resources are readily available at the starting times of appropriate critical activities. The study further concedes resource buffers being wakeup calls provide regular updates to the critical resources of their impending critical assignments.

It is subjected to the criticism in Barnes et al. (2003) that, number of feeding and resource buffers which address scheduling and monitoring rationale in a manner varying from other activities, are likely increase the tenderness of the schedule being complicated.

2.5 Presumptions of Human behavior in relation to CCPM

"TOC project management, on the other hand, attempts to account for certain typical human behavior patterns during project planning and execution" (Steyn, 2000, p.364).

Unveiling the human temptation in adding contingency reserves in to the individual activity durations, Steyn (2000) noted, people are used to be more complaisant on downside risk of delays rather than the upside opportunities for early completion because there is less incentive for completing an activity early though its opposing results are more times severe.

"People estimating activity times for a project usually believe that the project manager wants low-risk activity times; perhaps a probability of 80% to 95% completion on, or less than, the activity duration estimates" (Leach, 1999, p.42).

"In most project environments, people feel good if they complete an activity by the due date, and feel bad if they overrun the due date" (Leach, 1999, p.42).

This indicates that, people who establish durations for the activities in projects are invariably more prone to add low-risk high probability durations.

Exemplifying Parkinson's Law and the Goldratt's student syndrome, Leach (1999) convinced the human temptation to wait until the works become extremely urgent wasting their contingency time prior commencing the work. This behavior leads very less or no extra time to be spent on unanticipated backdrops and would eventually end up in a time overrun.

Substantiating the fact, works expand to fill the time available Steyn (2000) discloses workers induced by the lack of incentives are used to centre their concern more on the quality of the work by spending time extensively on outperforming the specification than completing the works reasonably early.

Delving in to facts which impede the project schedules to pass on positive variances, Steyn (2000) disclosed human custom to drive extensive review of the completed tasks during the additional time left from an early activity completion and their reluctance to admit the defective early duration estimates or to breach the credibility retained in doing so, are in the midst of adversarial causes.

Leach (1999) mentioned, in a project environment there is hardly or no significant incentive for an individual to complete an activity early, conversely people receive penalties for being late or having quality issues, or get more works to perform following the early completion. These project cultures are stated as inducing individuals to perform activities not early than the scheduled date.

"People actually perform in a multi-activity mode by dividing the time between multiple activities" (Leach, 1999, p.44). Thus, multitasking is performed by people in such a way that none of the outputs could submit before total of the time duration for all multiple activities.

Kokoskie (2001) submitted facts for multitasking as an unwise way of planning, which ultimately leads human behavior towards a declination in productivity. It is pointed out rapid mental changes that happen switching in to tasks at shorter intervals at physically different work location or works different in nature and working little time on all parallel tasks tend to decline the productivity.

"The importance of the assumptions regarding human behavior during project execution is similar to the importance of assumptions regarding behavior during planning" (Steyn, 2000, p.366). It is further illustrated; optimism which comes in to existence at the human mindset during the performance stage wastes the extra contingency reserves which were added by the pessimism prevailed at the planning stage. Waste of contingency reserves are reasoned by the fact; spending time more on external activities subjected to multitasking at the beginning of the planned activity duration.

Barnes et al. (2003) draws a hypothesis which likely exist in human behavioral aspects that runs counter CCPM approach. The Conversance persistent in the human notion that, initial duration estimates are likely to be curtailed, prompts them to initially submit more extensive duration estimates. This is claimed to be done upon the belief, following the reduction safety would still be left. Albeit, the study questions whether people would willingly agree to curtail the durations that they estimated for the individual activities supposed to be performed by themselves.

2.6 Buffer management process

"The risk of a delay is measured by the extent that the buffers have been consumed. Therefore project buffer and feeding buffers should constantly been monitored" (Steyn, 2000, p.368).

> On the concept of reporting, instead of detailed status or earned value reports, critical chain reporting consists of statistical process control-like checks of expected buffer consumption. Because of this focus on buffer consumption as the primary means of monitoring the project, the technique is sometimes also called critical chain buffer management. (Kokoskie, 2001, P.4)

Table 2.1: Buffer penetration thresholds and recommended actions

Buffer Consumed	Recommended Action
<1/3	None; within statistical control
1/3 to 2/3	Develop action plan
>2/3	Implement action plan

Source: Kokoskie, 2001

Actual time implications of the activities are traced and compared against the plan. Table 2.1 depicts the action plan as the buffer consumption reaches each fractional milestone. But this recommended action plan seems oblivious with regard to the proportional elapse of time from the planned schedule, against which to examine and drive controlling measures as above. This defectiveness is reformed by the illustration given in figure 2.2. Geekie and Steyn (2008) demonstrate that trigger lines disperse nearing the project closure depicting management actions are necessitated more acutely at the beginning. This is in line with the explication by Leach (1999) as works expand to fill the time available less work is done at the latter part of the project duration requiring less management actions with the progress.

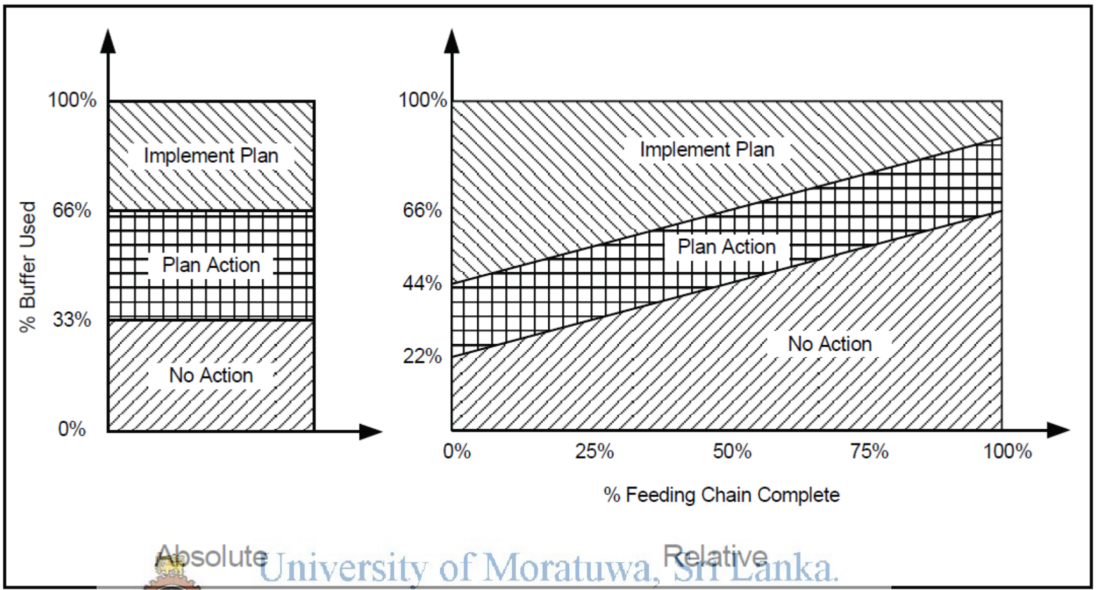


Figure 2.2: Trigger points in buffer monitoring
(Source: Leach, 1999)

According to Barnes et al. (2003) CCPM underlines that priority to be more exerted towards the activities within the chains which are more susceptible in their corresponding buffer penetration. Assigning constrained resources in to activities this rationale is stated as worth taking in to account, though ones with high penalties for delaying or with key strategic values are customarily prioritized regardless of the rate of buffer penetration.

2.7 Buffer sizing techniques

2.7.1 Cut and Paste Method (C&PM)

Goldratt (as cited in Leach, 1999) half of the project activity duration estimates as the buffer size ushered in CCPM. Nevertheless, CCPM buffer sizing method is relegated by Barnes at al. (2003); "CCPM does not provide any scientific or objective basis for determining the buffer size". Geekie and Steyn (2008) claimed this method shortens the overall project duration by 25%.

Eksioglu et al. (2005) exemplified the buffer proposed in C&PM as the half of the sum of 50% safe time estimates calculated from the activity path.

"After critical chain has been determined, the next step is to sum all the safety cut from the feeding chain and then take half of this sum and use it as feeding buffer" (Eksioglu et al., 2005). Nevertheless it is further stressed out; buffer is likely to increase unreasonably with the length of activity path resulting unnecessarily larger buffers, at the same time it derives inadequately shorter buffers in shorter activity paths. Aryanezhad, Ashtiani and Fallah (2010) commenting on C&PM notes "size of buffer increases linearly with the length of activity chain".

Geekie and Steyn (2008) unveil a disadvantage of this method as; it inherently constructs the buffer disregarding known variances in activity duration estimates. C&PM is similarly subjected to the critique drawn by Liu et al. (2000) as, 50% duration cut possibly reduce certain activity durations unreasonably, hence the study proposes identical cutting ratios for each activity.

2.7.2 The Root Square Error Method (RSEM)

Newbold (as cited in Eksioglu et al., 2005) formulated uncertainty in activity durations as,

$$U_i = S_i - D_i,$$

Where, *i* is the task, *Ui* is the uncertainty of the task, *Si* is the safe estimate of the task duration, *Di* is the average (50%) estimate of the task duration.

It is thereof introduced a statistical mean for the buffer as, the sum of two standard deviation aggregated from each activity duration estimate. Whereas, the standard deviation for activity duration is defined as, Standard deviation = sqrt [$(U_1/2)^2 + (U_2/2)^2 + \ldots\ldots\ldots\ldots + (U_n/2)^2$]

(sqrt; square root)

Hence,

Buffer size = 2σ = sqrt [$U_1^2 + U_2^2 + \ldots\ldots\ldots\ldots + U_n^2$]

"Compared to C&PM, RSEM has a distinct advantage of not generating very large or very small buffer sizes based on length of the feeding chain" (Eksioglu et al., 2005).

Geekie and Steyn (2008) define RSEM as the "square root of the sum of the squares (SSQ) of the difference between low risk duration and the mean duration for each task along the chain leading to the buffer". Accounting probable variations in task durations and underestimating the buffers in longer chains are respectively an advantage and a disadvantage in this method as convinced by this study.

2.7.3 Bias plus RSEM

Bias as urged in Geekie and Steyn (2008) is a factor which makes the project implementation longer but at any rate not shorter than the plan. It is urged as a comparatively smaller fixed portion to be added to the RSEM buffer.

Leach (2005) presented following guidelines in sizing the fixed portion of the buffer triggered by bias.

Table 2.2: Guidelines for sizing fixed portion of the buffer

Cause of bias	**Range of the buffer size**
Omissions	Not specified
Path merging (more than 5 parallel paths)	Up to 20%
Errors	5%-20%
Special cause variation	0%-30%
Failure to report necessary rework	0%-20%

Source: Leach (2005)

Geekie and Steyn (2008) shading lights upon the above states, experience shall have to be exerted in computing the fixed portion of the buffer complementary for the RSEM.

Observing in the presence of bias activity path mean duration being a better representation of the buffer Geekie and Steyn (2008) concluded buffers shall consist of a fixed portion which is proportional to the mean path duration and a variable portion proportional to the standard deviation. Hereunder, the fixed portion is advised to be less than 50% of the path duration estimate. It is further advocated to establish this fixed portion referring to a database formed with the record of statistics of the organization in performing previous project schedules.

Trietsch (2004) in another study challenged the assumption of independence of the activity duration estimates which is subjected to the RSEM. The study pointed out the said assumption would lead lengthier chains with higher number of activities to generate negligible buffer estimations. It is thereof presented a model to evaluate systemic errors or biasness that effect in buffer sizing and establishes lower limit on the buffer size to prevent the buffer become relatively small in lengthier chains.

2.7.4 Adaptive buffer sizing procedure with resource tightness (APRT)

Eksioglu et al. (2005) submits a buffer sizing approach based on resource tightness calibrated by Resource utilization Factor (RF). Hereunder, resource tightness presents as a yardstick for project characteristics and uncertainty.

"When the total resource usage is close to the total resource availability, it is more likely that delay will occur. Thus there should be larger buffers to absorb the delays" (Eksioglu et al., 2005). RF of a resource is defined as the ratio between resource usage and resource availability. Buffer sizing is formulated there as,

$RF(q) = (\sum_i r(i,q) * d_i)/T * Rav(q)$

$r' = \max_q \{RF(q)\}$

$K = 1 + r'$

$SUM = \sum VAR_i$

Buffer size = K*sqrt(SUM)

r(i,q); Resource usage for the activity i for resource type q, d_i ; Duration of the activity i, Rav(q); Availability of the resource type q, T is the length of the critical chain, VAR_i is the variance of the activity duration i.

2.7.5 Relative Dispersion (RD)

RD as a measure of uncertainty in activity duration estimates, is ushered in Shou and Yeo (as cited in Geekie & Steyn, 2008) and is formulated as below.

$$RD = \frac{\sigma}{t_e}$$

σ = Activity's standard deviation

t_e = Activity's mean duration

The former study makes classifications to activities based on their inherent uncertainties. The study goes on assigning tentative range of durations in to activity buffers applicable for each respective A, B, C and D activity classification. Said buffers are presented fractionally out of their mean activity durations as table 2.3 below. It is further tangible, the uncertainty as scaled by the classification presents in Table 2.3 is susceptible to increase as classification descends A to D. Nevertheless, the study fails to draw any coherence in between RD and the proposed activity buffers respectively for each classification.

Table 2.3: Buffer assigned to each activity classification as a percentage of the mean activity duration

Classification	Low safety	Median safety	High safety
A	4%	8%	12%
B	12%	24%	36%
C	20%	40%	60%
D	28%	57%	85%

Source: Shou and Yeo (as cited in Geekie & Steyn, 2008)

Replenishment to above shortcoming is made by Aryanezhad et al. (2010). The study raises five level risk classifications to activities based on their standard deviations applicable for activity durations (Table 2.4). A similar classification is also supposed to be realistic if RD is attributed in lieu of the standard deviation.

Table 2.4: Classification of activity risk subject to their standard deviation

Risk	Standard deviation
Very low	0.1
Low	0.2
Medium	0.3
High	0.4
Very high	0.5

Source: Aryanezhad et al. (2010)

Therefore Table 2.3 and Table 2.4 being complimentary for each other derive uncertainty classification criteria with regard to the variability potential in activity duration estimates.

Geekie and Steyn (2008) examine in the absence of biasness, the behavior of buffer against the mean and the standard deviation of activity paths. The investigation is done via a Monte Carlo simulation concluding with the underlying note.

Buffer size solely depends on the standard deviation of the activity path duration, computed attributing law of aggregation towards individual activity durations and central limit theorem upon their integrity in forming activity path duration.

The presiding study experiencing buffer penetration being linearly correlated with activity path standard deviation at the same time it is independent from path mean duration, advice not to size buffers based on their mean activity path durations.

Nevertheless, following a further examination by Geekie and Steyn (2008) claimed in the absence of biasness buffer penetration as a proportion is linearly related with RD. Figure 2.3 illustrates the relationship between simulated results obtained under the said study for mean buffer penetration and the RD.

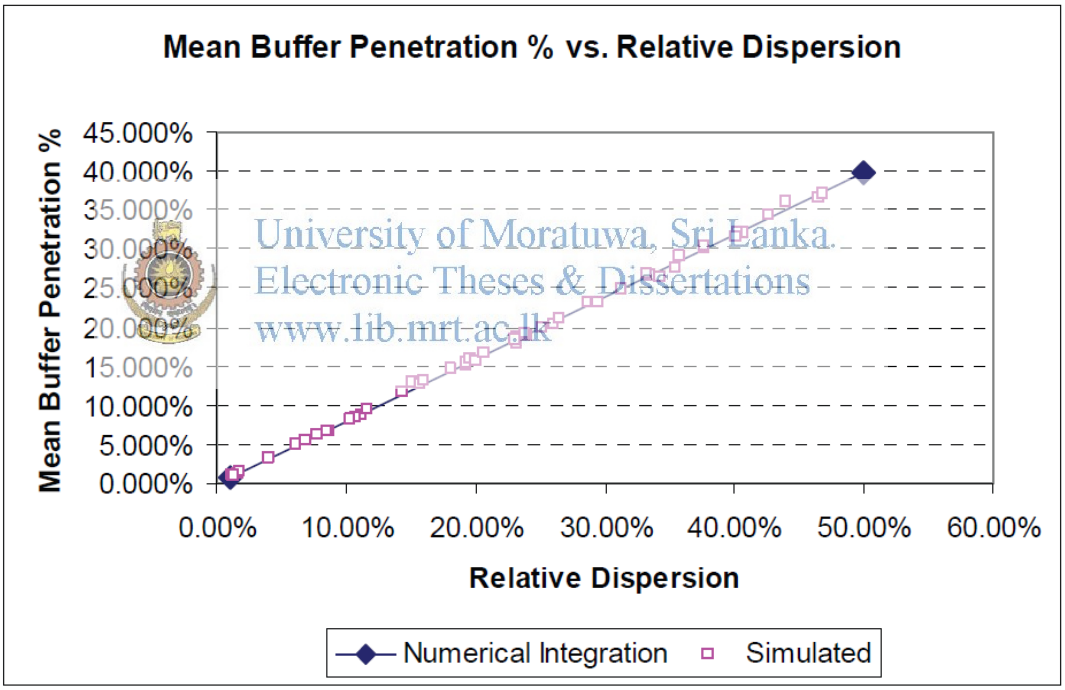


Figure 2.3: Relationship observed in between mean buffer penetration and RD (Source: Geekie & Steyn, 2008)

Figure 2.3 further appears conveying an intuition to the measure of biasness, which has gone unnoticed to its founders. The intensity of biasness in an activity path should be inferable from the positive fluctuation of the residual error observable in between the mean buffer penetration and RD. Positive fluctuation that presents in the residual error interprets the tendency at that RD value to have a greater mean buffer penetration over the generalized near linear relationship. Hence, complying with the foregone definition given to activity path biasness, positive fluctuations in residual errors that present in the above relationship render a scale for the bias measure.

2.7.6 Difference between left hand side and the right hand side of the distribution

Geekie and Steyn (2008) introduced another approach for buffer sizing. “Mean buffer penetration percentage is equal to the difference between the mean of the right hand side and the mean of the left hand side of the normal distribution feeding the buffer, divided by the overall mean of that distribution, and expressed as a percentage”.

The presiding study presents the underlying equation to calculate the means of both the right and left hand sides of a distribution. Here the lower and upper limit at six standard deviations is to replace the positive and negative infinity durations appropriate with the distribution.

$$\mu = \frac{1}{\sigma\sqrt{2\pi}} \int_{a}^{b} x.e^{\frac{-(x-\mu)^2}{2\sigma^2}}.dx$$

X = Activity duration plotted at the considered side of each single data point in activity duration distribution

σ = Standard deviation of the activity duration distribution

μ = Mean duration of the activity duration distribution

a = Lower limit of the numerical integration, calculated six standard deviations less the mean for left hand side of the distribution and the mean for right hand side of the distribution

b = Upper limit of the numerical integration, mean for the left hand side of the distribution and six standard deviations greater than the mean for right hand side of the distribution

2.7.7 Buffer sizing procedure proposed in enhanced TOC

Liu et al. (2000) in order to determine the percentage acquisition of the buffer from critical chain duration formulates a ratio as outlined below. The study hereunder does not consider the variations in activity duration distributions.

Percentage of buffer acquisition = (T1/T2) x 100%

T1 = Unconstrained critical chain duration

T2 = Constraint critical chain duration

Each activity's duration is proposed reducing by this percentage and the ultimate project buffer is to be sized to the accumulated reductions in durations.

Uncertainty in duration estimates where the CCPM sought solutions by adopting buffers does not seem to have satisfied with this study. Establishing a buffer proportional to the difference between the resource constrained duration and the unconstrained duration further appears irrational. Once activity durations are curtailed from the resource constrained schedule resource contention is again likely to become intriguing.

Elmaghraby et al. (2002) drawing contentions in counter to the findings by Liu et al. (2000) and making divulgence to its irrationality stressed, the inherent risk of variability within the activity durations is unlikely to present any coherence with resource constraint. The critique further broadens as the study by Liu et al. (2000) goes unnoted in its adopted method to give any reference to time-cost or time-resource trade off problems.

2.7.8 Adaptive risk based procedure (ARBP)

Upon the hypothesis, activity durations are random variables scattered in a lognormal distribution function Aryanezhad et al. (2010) formulate a buffer sizing approach based on its principle uncertainty indicators assigned to activity durations.

"In lognormal distribution with right skewness, the value of mean would be more than median's" (Aryanezhad et al., 2010). Defining "time indicator of risk of activities" the study denotes, with the increase in risk inherent, the standard deviation of the activity duration would also increase. This effect is further enlightened as liable to increase the difference between the mean and the median values i.e. activity risk indicator.

Activity risk indicator = (Mean – Median) of activity duration distribution

The presiding study presents "uncertainty indicators" as codified below.

(a) Coefficient of variation

$$\textit{Coefficient of variation} = \frac{\sqrt{Variance}}{Mean}$$

(b) Skewness

"This indicator is used to measure the lack of symmetry of a distribution, so when the measure of skewness increases, the distant numbers have more distance from the mean" (Aryanezhad et al., 2010).

$$Skewness = \frac{E\left[(X-\mu)^3\right]}{\sigma^3}$$

X - Activity duration

μ - Mean duration

σ - Standard deviation

(c) Kurtosis

The study demarcates kurtosis as the flatness or the peaked nature of a lognormal distribution. With the rise of the kurtosis activity ingrained risk is likely to increase.

$$Kurtosis = \frac{E\left[(X-\mu)^4\right]}{\sigma^4}$$

Aryanezhad et al. (2010) broadening its examination subject to uncertainty and its inference of lognormal distribution function with regard to the activity durations forms the below equations in order to size the buffer.

- Equation (I): Project buffer = Coefficient of variation X Activity risk indicator
- Equation (II): Project buffer = Skewness X Activity risk indicator
- Equation (III): Project buffer = Kurtosis X Activity risk indicator

2.8 Resource constraint project scheduling under CCPM

"An acceptable way to account for potential resource constraints on the project is to first identify the critical path and then perform the resource leveling" (Leach, 1999).

Steyn (2000) criticized CCPM concept of implementing resource leveling prior establishing the critical chain. It points out resources leveling causes the non-critical activities to be executed with minimum resources making them resource tight and enabling their resources to be busy for the entire duration of the project. An argument is thus presented; resource leveling contravenes the requisite under CCPM that non-constraints shall subordinate to the exploitation of the constraint.

Eksioglu et al. (2005) points out that, there is no specific procedure suggested by CCPM to resolve resource contention. Leach (as cited in Eksioglu et al., 2005) suggested resolving resource contention only for most conflicting tasks and for such conflicting tasks connecting the critical chain at the nearest points to the project end. Herroelen and Leus (as cited in Eksioglu et al., 2005) suggested exclusion to the resource conflicts by means of Resource Constraint Project Scheduling Problem (RCPSP) heuristics. Therefore resource constraint is allegedly not an unraveled issue by CCPM, even though exclusion of the same is highly recommended in its ideology.

Liu et al. (2000) with its enhanced TOC methodology substitute resource buffer suggested in CCPM with a resource time flag. The study explains resource time flag as an in advance duration for the critical resources to complete their current task and then to be ready with the next critical task. Resource time flag i.e. advance readying time for critical resources, gives a time implication to the resource buffer. Resource flag times added to the outset of task durations are to be sized considering each activity's resource constrained time implication less their constraint free schedule duration. The enhanced TOC is constituent of a heuristic approach based on set priority rules, in order to determine the resource constrained critical chain duration.

2.9 Contribution of literature in research problem

The foregone analysis of conceptual information with regard to CCPM and associated theories brings forward the following identified lapses which are contributory for the research problem.

- Arbitrary fractioning of contingency reserves within task duration estimates in sizing project buffers
- Existing buffer sizing approaches are independent from scheduling constraints (ex. Resource constraints)
- Project buffer estimates are not optimized among all available values at a one scenario

2.10 Summary

The chapter brings about the three (03) theories of CCPM as the mere substance of CCPM ideology. The three theories manifest,

- The system constraint to be the centre of exploitation.
- Common cause variations i.e. statistical deviations, distinguishing from special cause variations are to be addressed in establishing project durations.
- Statistical law of aggregation which renders that, accumulated variations in individual tasks durations form the potential variation to the project duration.

CCPM elucidate the following six (06) undesired effects that lead for project failures due to uncertainty.

- Excessive activity duration estimates due to safety reserve ingrained within the activity duration estimate which is likely to waste.
- Expansion of time for physical performance to fill the entire time allocation within the project schedule for the activity.
- Failure to pass on positive variations by early task completion.
- Merging of feeding paths in to the critical path due to longer durations acquired by feeding path activities.

- Performing multiple tasks by the same group of resources which likely to delay all the multiple tasks simultaneously performed.
- Loss of project management's focus due to the misconceptions developed by; early starting times of the tasks, diversion of project critical path and superstition on earn values.

The chapter plunges to elaborate the steps including the followings to curb the foregone undesired effects for exclusion.

- System constraint to be identified as the critical chain resultant by activity dependencies and resource dependencies.
- The sequential procedure constituted in CCPM involves project buffer to be sized as the difference between its initial low risk high confidence estimate and the sum of 50% confident i.e. the median duration of task duration distributions.
- Feeding buffers and resource buffers are to be positioned in order to secure the project critical chain from changes due to activity path merging and resource contentions.

Human behavioral aspects giving way to undesired effects for project failures are explained by the chapter.

Buffer management procedure which rules out buffer consumption by actual time implication of task performance is explained referring to the principles set out in CCPM.

The chapter further to present alternative buffer sizing approaches available in literature and resource constrained project scheduling as supplementing the purviews of CCPM.

CHAPTER 03

RESEARCH METHODOLOGY

3.1 Introduction

The chapter discusses the approach driven for establishing the proposed buffer sizing approach. It is further elaborated under the chapter; the methods and types of conceptual data collection in relation to CCPM and FIS ideologies and in relation to the empirical test on validating the proposed approach. Data analyzing techniques in order to convey the above aggregated conceptual data in to a stepwise buffer sizing approach are also disclosed within the chapter.

3.2 Research approach

Following the identification of the research problem and following the retrieval of conceptual information with regard to CCPM and FIS, stepwise buffer sizing approach is formulated. Hereunder, CCPM and FIS furnish conceptual data in relation to the substances of the research i.e. principles under buffer sizing and technique of analyzing principles respectively. Subsequently the proposed approach is conceptualized in steps as the solution to the existing problem and validated for its pragmatism with a case study. Hence, the study follows an inductive research approach in addressing its aims objectives.

3.3 Data collection: CCPM concepts of buffer sizing

Formulating the new approach for sizing project buffer conceptual data comprising theories, principles and techniques are drawn from CCPM ideologies explained in litrature. Nevertheless, the approach portrayed under the study has certain distinctions varying from CCPM concept brought about in literature review.

3.3.1 Buffer sizing

Drawing coherence to the school of CCPM the proposed approach is conceptually confined only to some of its principles extending reasonable context to the buffer sizing in water distribution projects. Certain assumptions and limitations to the study are postulated within CCPM theories. In this phenomenon three theories and CCPM planning process and six undesired project effects to the exclusion elaborated in the presiding chapter are more or less connected to the current study.

Nevertheless, varying from CCPM as a project management strategy the current study converges only towards codifying an approach for sizing project buffer. In this respect sizing and positioning of a project buffer as advocated in CCPM theories reflects a cognitive image of an optimistic project schedule as a resolution to the prevailing problem in scheduling water distribution projects.

3.3.2 Exploiting the system constraint

The study centers and cures only the system constraint within the project schedule. Hence it focuses only on identifying and exploiting critical chain of a project. The proposed buffer sizing approach thus rehabilitates the sizing of project buffers which are added at the end of critical chains. But, the study is devoid of conceptualizing for project feeding paths and appropriate feeding buffers. Nevertheless, even though the study does not suggest streamline the resource buffers, resource constrained project scheduling is considered whereas critical chain is in its central focus.

3.3.3 Common cause variations

Similarly as CCPM advocates the study is contained to common cause variations which results for statistical deviations in activity paths. Special cause variations initialized by factors of project characteristic values are out of analysis.

3.3.4 Arbitrary apportionment

CCPM derives its median estimate at 50% confidence via probability distributions drawn to activity duration estimates accounting for common cause variations. Nevertheless, this appears to be a generalized abrupt apportionment exerted for task duration distributions having no other exceptional justifications than being an averaged value. The same is incapable in adopting for certain scheduling constraints. Hence the new approach is designed deviating from any arbitrary fractioning for sizing buffers that submitted in certain literature.

3.3.5 Optimistic but realistic durations estimates

As Leach (1999) standardizes "contingency reserves" within activity durations, optimistic duration estimates are doubted as unreasonably aggressive when they are free from such contingencies. Jan and Ping (2006) and Steyn (2000) positing views on CCPM contingency reserve emphasize safety free duration estimate to be realistic. Similarly to the later notions present buffer sizing is drawn by leaving optimistic duration as a reasonably minimum estimate.

3.3.6 Presumption of Student syndrome (Parkinson Law)

Student syndrome is presumed as a realistic empirical reason to ruin the safety times housed inside task durations. This is conducive due to the common aspects of human behavior. Student syndrome caused by redundant safety times makes the task performers to be relaxed and be induced to expand the time incurrence to fill the time available. Since this effect is pursuant by intangible reasons, it is to be inferred by imprecise and incomplete information. All known about student syndrome is that, the larger the inbuilt safety, the larger the potential for its effects.

3.3.7 Sanction to activity path merging

The proposed buffer sizing approach is created assuming critical chain is motionless. It is understood when canvassing intellectual views at the outset of the study, that critical chain of a water supply project is customarily subjected to perpetual diversions. Hence as CCPM assumes an immobile critical chain seems out of pragmatic concern. But the present study is calculated to cater the needs initialized by the variations that happen to critical chain.

3.3.8 Resource constraints

The proposed buffer sizing procedure is formulated safeguarding the resource constraints imposing upon the project schedule. Activities are accordingly to be scheduled within the boundaries of resource availabilities. A specialized crew to perform each recurring task is supposed considering the practices rooted in water distribution projects. Therefore each recurring task within the work breakdown structure of the project schedule is customarily an assignment for a one specialized crew. Therefore, time-resource trade off problem is not a matter of concern under the new approach. The new approach is designed to optimize; the safety exempted task durations with comparatively smaller buffers of where resources are profusely available and vise versa where resources are in scarce. Hence resource leveling is not the supposed way of addressing resource contention. Nevertheless, for the simplicity the approach is developed assuming that resource availabilities are constant during the entire duration of the project.

Since the proposed approach concentrates only on critical chain as the foremost activity path being exploited, supplementing the non critical paths becomes a sideline. Therefore, the study accounts only on catering the critical chain with existing resource availabilities. At this drawn limitation, multitasking ushered in the theories of CCPM declines to retain its interest under the current research.

3.3.9 Precedence relationships

Precedence relationships between tasks are accounted to be secured whilst sizing the project buffer. Buffer contribution from each critical recurring task under the proposed method is to be obtained allowing at least the minimal task duration for the non critical tasks. These non critical paths connecting the critical chain curtail at the same time critical chain shortens. Therefore whilst shortening the critical chain it is essential to leave not less than the minimal path lengths for the non critical paths as per the statistical observations to task durations.

3.3.10 Enabling revisions to buffer

The proposed approach does not incorporate a buffer management procedure as CCPM is constituent of. Instead the new approach proposes a revised buffer size considering the resources availability of the tasks and balance project duration at different times during the project duration.

3.4 Data collection: concepts of FIS as the analyzing technique

The proposed approach exerts Fuzzy Inference Systems (FIS) as the analyzing tool. FIS is switched due to underlying reasons and its own properties which are disserving for solving the problem of project buffer sizing.

3.4.1 Inefficiencies in probabilistic reasoning

Setting up of optimistic task durations at 50% confidence median duration estimate as delineated by Leach (1999) is of some urged suspensions. Project buffer is sized in to one standard deviation of the task duration distribution summated in accordance with the law of aggregation. This heuristic method proposed in CCPM upholds due to no other reason than being a generalized value. It is seemingly an arbitrary apportionment invariably fixed irrespective of project time related dynamics. CCPM put forward no recommendation for its median estimate over any other statistical fractioning (ex; mean, mode or such other specific percentiles). Varying from probabilistic reasoning FIS is based on possibility theory standing a solution for the prevailing backdrop.

3.4.2 Incomplete and imprecise information

A. Alola, Alola and Tunay (2013) underscore Fuzzy set theory as a mean of formulating incomplete and imprecise information at the same time that cures imprecise probabilities. The study elucidates how Fuzzy set theory brings in to light, the subjective knowledge fraught with incomplete and imprecise information leading for linguistic judgments. This extends supportive contexts to the present study as in fact optimistic project duration as the prerequisite for sizing project buffer appears a subjective determination to be generated by linguistic statements.

3.4.3 Unsolicited factor examination

A. Alola et al. (2013) criticize extensive examination of unknown and contingent factor analysis as probabilistic methods customarily exert time and effort. The study denotes FIS avoids the necessity of analyzing such ill known factors stuffed with data conflicts, imprecision and linguistic (statement) errors.

Therefore FIS does not warrant for project characteristic factor examination as the same way CCPM confines its buffer sizing only in to statistical deviations free from factor analysis.

3.4.4 Subjective and objective knowledge

Dubois (2006) drawing distinctions for subjective and objective realizations signifies, lack of knowledge gearing for subjective views cannot be modeled by similar methods as modeling objective views that reasoned by the variability of observations.

Certain possible events where observations are often made unsatisfactory in terms of variability of hardly measurable parameters or such events possessing incomplete knowledge or only limited to expert knowledge as pointed out by Dubois (2006) straight forward relates to the present study. Variables such as student syndrome under the present study and contentious optimistic project duration submitted in CCPM are of this nature. Dubois (2006) presenting his view on such criterion goes on to mention "such knowledge is generally not rich enough to allow for a full-fledged probabilistic modeling".

Foregone substantiation comprehends the weight of subjective viz. incomplete or expert knowledge to be processed in the proposed new approach which is facilitated by FIS.

3.4.5 Propagation of uncertainty

A. Alola et al. (2013) unfolding subjectivity, flexibility and plausibility properties of the concept of fuzziness states; though its less restrictiveness and simplicity gains cons in mathematics, the same features enables the concept to propagate uncertainty of expert systems. The researchers further goes on to praise as it is "more accurate in handling uncertainty of data impression that is void of randomness".

This specific distinction in FIS blended with inherent simplicity enables the new approach to gain its confrontational abilities against uncertain task duration estimates.

3.5 Data collection: Empirical test

The proposed approach is validated for its pragmatism with a case study executed with a set of water distribution projects complying the profile of projects forming scope and limitations of the research. The case study is limited to the underlying exhaustive set of dependant events which are recurring within project work breakdown structures.

- Laying of unplasticized Poly Vinyl Chloride (uPVC) pipes, fittings, valves to suit and accessories to distribution mains
- Laying of Ductile Iron (DI) pipes, fittings, valves to suit and accessories to distribution mains
- Culvert and bridge crossings
- Chamber construction and connection to the laid distribution mains
- Pressure testing
- Flushing and disinfection
- Reinstatement for the damaged carriageways

A critical chain of a water distribution project constituted with the above set of recurring tasks is chosen as the case study project schedule. Sample statistics of task duration estimates for each above recurring tasks are retrieved from a set of another water distribution project schedules. These statistical information are subjected to the analysis under case study as explained in forthcoming sections.

3.6 Data analyzing tools

Conceptual data gathered above as the substances of the proposed approach are then configured in to a stepwise buffer sizing approach using the following analytical methods.

3.6.1 Possibility theory

Possibility theory appears the very substance of FIS. Dubois (2006) explains that, possibility theory facilitates FIS in handling incomplete and imprecise information including reasoning with extreme probabilities whilst sanctioning for imprecise probabilities, appearing as a coarse nun-numeric version of probability theory it provides graded semantics to natural language statements and forms a platform to model partial belief.

Tsoukalas and Uhrig (1997) state possibility theory in knowledge analysis does quantification for meanings rather than measures of information. They further to mention in contrast with probability-possibility consistency principles founded by Zadeh (1978), "Possibility theory focuses more on imprecision intrinsic in language, whereas probability theory focus more on events that are uncertain in the sense of being random in nature"(p.38).

Possibility theory according to Dubois (2006) is consistent of the following duel set functions,

- Possibility measure; represents the maximum of degrees in disjunction of the events. This forms maxitive operator for the possibility theory.
- Necessity measure; represents minimum of the necessity degrees in the conjunction of events. This forms minitive operator for the possibility theory.

A. Alola et al. (2013) drawing context from probability-possibility consistency principle denominates possibility and necessity functions as upper and lower bounds of probabilities on the interval [0,1]. Zadeh (1978) at the advent of fuzzy set theory viewing on probability-possibility consistency principle comments "rather it is an approximate formalization of the heuristic observation that a lessening of the possibility of an event tends to lessen its probability-but not vice versa. In this sense, the principle is of use in situations in which what is known about a variable X is its possibility-rather than its probability distribution"(p.10).

The above principles form the way for the current study and given direct pursuant in delineating possibility and necessity variants in relation to task duration distributions absorbed from statistics.

A. Alola et al. (2013) and Dubois and Prade (2011) unveil these duel set functions and give them a qualitative facet as below.

"A" represents an event of a member in the universe "S" composed of the element "s". Then the proposition "s is A" can be represented as $s \in A$.

Possibility distribution (symbolized by π) depicts A's dispersion of possibilities over S in an ordered scale of a unit interval [0, 1]. π indicates the distribution of subjective knowledge of possibility of happening event A denoted by $\Pi(A)$ whereof its necessity is denoted as $N(A)$.

Therefore recalling that possibility and necessity are respectively of maxitivity and minitivity properties,

$$\prod(A) = \sup_{s\in A} \pi(s) \quad \text{Equation: 3.6.1. (1)}$$

$$N(A) = \inf_{s\notin A} 1-\pi(s) \quad \text{Equation: 3.6.1. (2)}$$

sup: Supremum inf: Infimum

Possibility and necessity interrelationship can be qualitatively expressed by;

$$N(A) = 1-A^c \quad \text{Equation: 3.6.1. (3)}$$

A^c: Complement of the event A

Considering maxitivity properties of possibility, disjunction of happening A and B events are represented by;

$$\prod(A\cup B) = \max(\prod(A), \prod(B)) \quad \text{Equation: 3.6.1. (4)}$$

Considering minitivity properties of necessity, conjunction of happening A and B events are represented by;

$$N(A\cap B) = \min(N(A), N(B)) \quad \text{Equation: 3.6.1. (5)}$$

According to the principles set out in A. Alola et al. (2013) membership function of the fuzzy set "A" within the universe of discourse "S" (s∈S) can be illustrated as;

$$\mu_A(s) = \pi(s)\ s\in A \quad \text{Equation: 3.6.1. (6)}$$

A. Alola et al. (2013) exemplifies possibility as the upper bound of probability. According to their findings possibility amidst its interval [0, 1] takes the value one if the event being considered can occur.

The above principles in association with possibility theory are then blended with other FIS related principles to explain the method exerted in building fuzzy sets for the identified parameters in deciding optimistic and pessimistic project schedules.

Tsoukalas and Uhrig (1997) differentiating crisp and fuzzy sets explained crisply defined elements of a classical set as crisp sets whilst fuzzy sets are of their degree of memberships [$\mu_A(s) \in (0,1)$] indicating the degree to which values of a variable belongs to the identified set. The study notes that, membership function can also be derived from statistical data.

Fuzzyfication is defined by Tsoukalas and Uhrig (1997) as a process of transforming crisp sets in to fuzzy sets. The study illustrates fuzzy set of the variable "A" as below. "A" acquires values on the universe of discourse "X" as "x" being its elements.

$$\mu_{A(x)}: X \rightarrow [0, 1]$$

Fuzzy set A = $\{(x, \mu_A(x))\}, \quad x \in X$ Equation: 3.6.1. (7)

3.6.2 Propagating subjective knowledge

Though, it is seemingly contentious drawing a straight line to represent subjective knowledge that governs project and task related variables, this is backed up in fuzzy set theory by Hait and Masmoudi (2012) as below.

> The strength of fuzzy modeling consists of respecting exactly what experts know about the uncertainty and no more. To be honest, small hypothesis are taken too while choosing the entire profile specially straight lines instead of curves. (p.3597)

The above simplification for propagating subjective knowledge is exploited in outlining task and project related variables under the new approach.

3.6.3 Fuzzy operators from fuzzy set theory

Fuzzy operators trigger the necessary cognitions between fuzzy sets. Ahmad and Akther (2009) and Tsoukalas and Uhrig (1997) present the standard fuzzy arithmetic operations. Adhering with such fundamentals and the operations to be implemented in the proposed approach following fuzzy operations become illustrious.

Let, A (a_A, b_A, c_A, d_A) and B (a_B, b_B, c_B, d_B) are two independent trapezoidal fuzzy sets belong to the universe of discourse X ($x \in X$)

$A \oplus B = (a_A + a_B, b_A + b_B, c_A + c_B, d_A + d_B)$ Equation: 3.6.3. (1)

$\text{Min}(A, B) = (\min(a_A, a_B), \min(b_A, b_B), \min(c_A, c_B), \min(d_A, d_B)$ Equation: 3.6.3. (2)

$\text{Max}(A, B) = (\max(a_A, a_B), \max(b_A, b_B), \max(c_A, c_B), \max(d_A, d_B)$

Equation: 3.6.3. (3)

$A \cup B = \max_{x \in X}(\mu_A(x), \mu_B(x))$ Equation: 3.6.3. (4)

$A \cap B = \min_{x \in X}(\mu_A(x), \mu_B(x))$ Equation: 3.6.3. (5)

$K \otimes A = (K.a_A, K.b_A, K.c_A, K.d_A)$ K is a constant crisp value Equation: 3.6.3. (6)

The above fuzzy operators are to be manipulated to form the aggregation between the task durations along the chain of activities in the proposed approach.

3.6.4 Alpha-cut arithmetic and aggregation method

Tsoukalas and Uhrig (1997) fabricate the synthesis of fuzzy sets along with the integration of ∝-cuts forming crisp sets consisting of the elements of fuzzy set to a minimum degree of membership of ∝.

According to the representation made via the presiding citation if A is a fuzzy set acquiring the elements of universe of discourse X ($x \in X$), then $A_\propto$ being a crisp set belongs to the fuzzy set A can be interpreted as,

$A_\propto = \{x \in X \mid \mu_A(x) \geq \propto\}$ Equation: 3.6.4. (1)

Equation 3.6.4 (1) denotes ∝-cut forming crisp set $A_\propto$ associates with the elements of fuzzy set A at least to the degree of membership ∝. Accordingly crisp set i.e. level set $A_\propto$ creates a level fuzzy set whose membership function $\mu_A(x) \geq \propto$ $[\propto \in (0,1)]$.

Thus, principles under Alpha-cut arithmetic operations are to be exercised in proposed approach in order to find the optimistic and pessimistic completion times of a task at various possibility levels.

3.6.5 Fuzzy cardinality measure and subsethood

Dhar (2013) throwing lights on the above posits if A ($x \in A$) is a subset of B ($x \in B$),

$A \subseteq B \Rightarrow \mu_A(x) \leq \mu_B(x)$

S (A , B) is degree of subsethood A in B

M(A) and M(A∩B) respectively cardinalities of A and A∩B

$S(A, B) = M(A \cap B)/M(A)$

$M(A) = \Sigma_i\ \mu_A(x_i),\ \forall\ x_i \in X$

$M(A \cap B) = \Sigma_i\ \mu_{A \cap B}(x_i),\ \forall\ x_i \in X$

Subsethood measures of task related variables within the project related variables are considered in the subsequent step for solving optimization problem in the proposed approach.

3.6.6 Non-linear optimization

According to the drawn explanations hereunder by Leavengood and Reeb (2002), decision variables are to be varied within the given constraint set until the objective function is adopted to the optimized solution.

Fuzzy subsethood to be optimized as the objective function by varying the task related variables. Nevertheless, in this respect project related variables appear as supersets to the task related variables and their availabilities form the necessary constraints to the subsets. This fundamental is utilized for making the rationale in computation to obtain optimized solution under proposed approach.

3.6.7 Data analysis under empirical test

The buffer is to be sized and positioned in the case study project schedule as the preliminary solution under the new approach.

Duration statistics for each recurring task undergoes the proposed stepwise approach in sizing project buffer for the case study project schedule.

With regard to the observations under empirical test, the proposed approach is validated for project buffer sizing in water distribution projects.

3.7 Summary

Profile of water distribution projects with an exhaustive set of recurring tasks which are commonly found within the formation of project schedules is constituted as the outline of limitation in constructing the rationale towards proposed project buffer sizing approach.

The chapter draws conceptual data retrieval from CCPM to form proposed project buffer sizing approach including,

- An approach only for project buffer sizing not for project management
- Addressing the project critical chain but not the feeding chains
- Restricting to the concern solely on common cause variations
- Drawing optimistic project durations which are necessarily realistic
- Presuming the Student Syndrome
- Giving resolutions when activity paths are merging
- Addressing resource constraints
- Allowing no violations to precedence relationships predetermined between the tasks
- Facilitating computation of a revised buffer when activity paths are varying instead of buffer management as submitted in CCPM

FIS as the vested technique of reasoning for data analyzing is justified within the chapter by weighing its pros over the cons against statistical reasoning techniques under the following headings. Following conceptual data are gathered in relation to the proposed analyzing technique vested in new approach.

- Inefficiencies in probabilistic reasoning
- Handling incomplete and imprecise information
- Elimination of unsolicited factor examinations
- Representation of subjective and objective knowledge is facilitated
- Propagation of uncertainty is facilitated

The chapter goes on explicating the below analyzing techniques which abide the conceptual data in to a stepwise sequential approach in the succeeding chapter.

- Possibility theory
- Propagating subjective knowledge
- Fuzzy operators from fuzzy set theory
- Alpha-cut arithmetic and aggregation method
- Fuzzy cardinality measure and subsethood
- Non-linear optimization
- Data analysis under empirical test

The empirical test is to draw final conclusions upon the empirical success of the proposed approach and its sophistication in water distribution projects.

CHAPTER 04

PROPOSED CONCEPTUAL APPROACH FOR SIZING PROJECT BUFFER

4.1 Introduction

The study sequentially portrays its proposed approach for project buffer sizing in steps below. This according to the principles set out under research methodology.

4.1.1 Step 01: Continuous Density Functions (CDF) of the probabilities of task durations

CDF of the duration estimates corresponding to a defined one unit of work of the identified set of recurring tasks are analyzed. A reservation of historical values for these variables is a prerequisite. Probability density at this juncture is the frequency of events between a range of variable values averaged by the number of total observations within the sample. Hence when sum of all the sample values multiplied with their respective frequencies corresponds to one, probability density can be illustrated by the area below CDF of the range of values concerned.

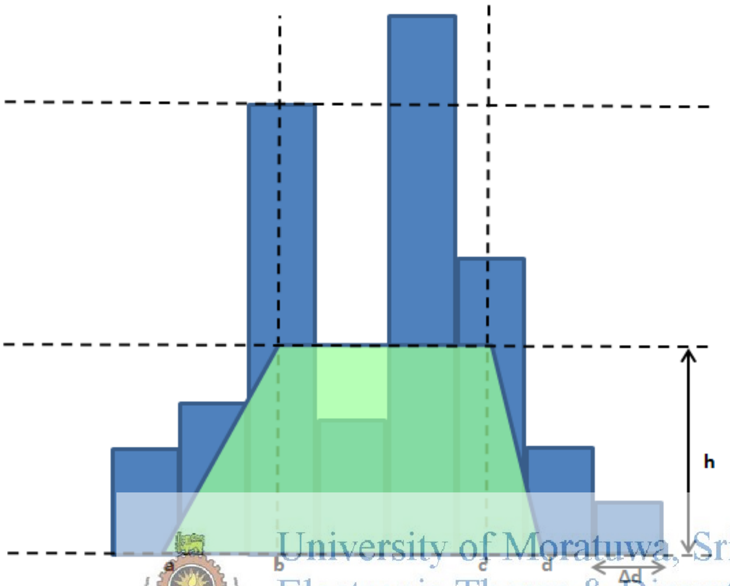
- Collected historical duration estimates to perform a one identified unit of work under a particular recurring task to be grouped in equal (N) number of intervals between their smallest and largest observed values.
- Histograms for the duration distribution of each recurring task are to be drawn considering their group mid values and respective frequencies.

If at i^{th} group of duration interval,

Relevant frequency of observations for the group = f_i

Hence, probability density of the group = $f_i/\sum_{i=1}^{N} f_i$

Equation: 4.1.1. (1)



a-Minimum of task duration estimate to perform its one unit of work at 95% confidence

d-Maximum of task duration estimate to perform its one unit of work at 95% confidence

-Group frequencies at each equal (N) number of group intervals (Δd),

-Represents the trapezoid;

(1) acquiring the area occupied by group intervals multiplied by their respective frequencies in between minimum and maximum task durations at 95% confidence

(2) at frequency level (h), b-c represents the interval between the most outward frequency columns of the histogram within the minimum and maximum limits of task durations at 95% confidence

Figure 4.1: Histogram of activity duration distribution and the trapezoid representing its distribution at 95% confidence.

Trapezoid (a, b, c, d) is obtained as its uppermost line (b-c) parallel to the base (a-d) is conjoining the mid points of the most outward frequency columns (at b and c variable values) at its vertical level (h). (a) and (d) are the smallest and the largest of the duration estimates obtained for the distribution at 95% confidence. (h), (b) and (c) values are thus selected as the area occupied by the trapezoid equals the area occupied by the histogram within the 95% confidence interval on duration axis.

If the area acquired by the trapezoid (a, b, c, d) is represented by the function f(t) then;

$$\int_{i=a}^{i=d} f(t)\,dt = \sum_{i=a}^{i=d} \Delta d \times f_i$$ Equation: 4.1.1. (2)

Then (h) is to be re-calculated whilst the area occupied the histogram equals to one. Similarly as for this, frequency attached with each group is required to be done, (h) needs to be divided by the number of sample observations (i.e. sum of frequencies of all the groups). Thence (a, b, c, d) trapezoid depicts CDF of the probability that task duration distribution possesses at 95% confidence level.

4.1.2 Step 02: Identification of the critical chain

Similarly as it is conceptualized in CCPM related literature critical chain is designated at this step as the resources leveled critical path. Whereas resource leveling is not in concern critical path becomes the critical chain.

Nevertheless prior critical path is denominated project activities are to be arrayed according to their dependency relationships in order to determine the longest sequence of paths.

The longest chain of activities is determined assigning the shortest durations to the tasks without violating their logical dependency relationships and resource dependencies. The shortest durations are the minimum durations at their 95% confidence obtained from appropriate frequency distributions of duration estimates observed for each task (similarly as obtained duration (a) as shown in figure 4.1 under the explanation for stage 01).

4.1.3 Step 03: Fuzzification of antecedents

Considering the application of principles under possibility theory and their maxitivity and negativity operators, the underlying project and task related variables are to be formulated in to appropriate fuzzy sets.

4.1.3.1 Trapezoidal fuzzy set for the task duration distribution

A fuzzy set is modeled for the duration distribution of a typical recurring task (denominated as DuRT(A)). For the purpose of this, trapezoid built in stage 01 depicting CDF of the probability that task duration distribution (for its one unit of work to perform) possesses at 95% confidence level is rationally examined.

Transforming the trapezoid representing CDF of probabilities in to a fuzzy set, its highest CDF value is given the membership value ($\mu_{DuRT(A)}$) one (i.e. h=1.0) whilst the smallest is given zero.

According to Equation: 3.6.1.(6) dispersion of $\mu_{DuRT(A)}$ on fuzzy set elaborates the distribution of possibilities ($\prod_{DuRT(A)}$) for the values that RT(A) takes for its duration.

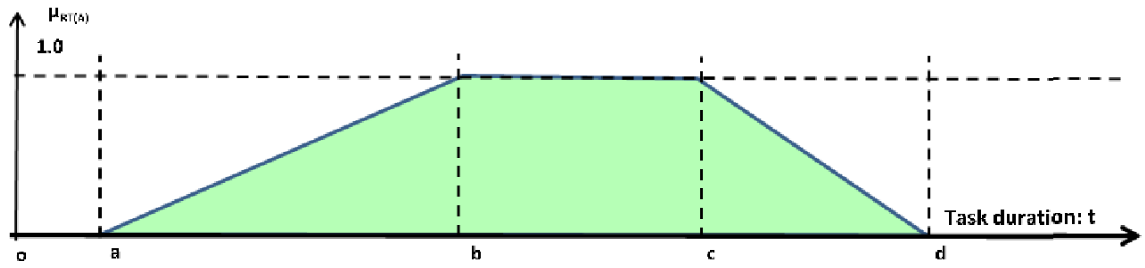


Figure 4.2: Trapezoidal fuzzy set representing the duration distribution to perform a one unit of work of a typical recurring task denominated as; RT(A).

As it is elaborated in equation: 3.6.1.(7) the above trapezoidal fuzzy set can be demarcated by the underlying trapezoidal fuzzy numbers which earmark the fuzzy restrictions imposing on RT(A)'s duration distribution.

Trapezoidal fuzzy numbers for typical recurring task RT(A) for its one unit of work to be performed is denoted by TraFN[DuRT(A)].

Hence,

TraFN[DuRT(A)]: (a, 0.0), (b, 1.0), (c, 1.0), (d, 0.0)

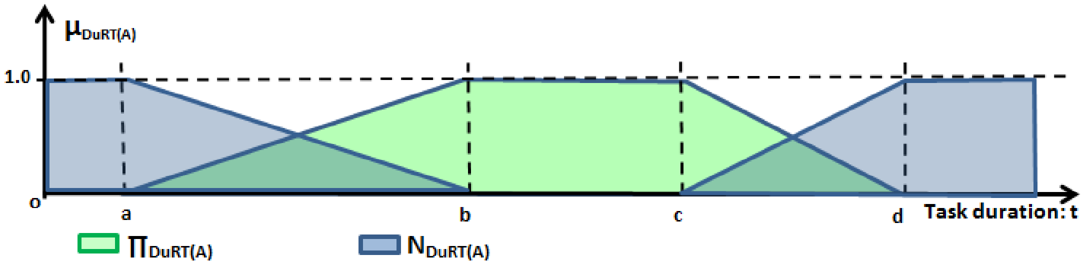


Figure 4.3: Possibility and necessity functions for the duration distribution for one unit of work under the typical recurring task

According to figure 4.3 the smallest duration to which RT(A) can be scheduled is (a). From duration time (a)-(b) $\prod_{DuRT(A)}$ increases whilst between the values (b) and (c) the respective possibility reach its highest. Thereafter, $\prod_{DuRT(A)}$ declines in between (c) and (d). Conversely minitivity function $N_{DuRT(A)}$ of task duration which implies the existence of RT(A) duration is maximum at one when $t \leq a$ or $t \geq d$ and minimum at zero when $b \leq t \leq c$. Respectively in between a-b and c-d $N_{DuRT(A)}$ gradually declines and increases as oppose to $\prod_{DuRT(A)}$.

4.1.3.2 Construction of fuzzy Guntt bar

Analogous to the fuzzy representation for the task duration estimate, fuzzy Guntt bar to be constructed taking in to account of the following facts;

- Minimum possible duration for RT(A) is (a)
- Possibility to exist the Guntt bar at a specific point of time within the project duration is its upper bound of probability to exist.

It is apt to be noted, Guntt bar elaborates the existence of the recurring task between the assigned starting and finishing times. Fuzzy Guntt bar in modeling this new approach, brings about the dispersion of possibilities and necessities for the starting and finishing times of recurring tasks along with the project schedule.

Assuming finishing time of the preceding activity is at time (k) and denominating the fuzzy Guntt bar for one unit of work of RT(A) as GunttRT(A), fuzzy Guntt bar for RT(A) can be illustrated as,

TraFN[GunttRT(A)]: (k, 0.0), (k, 1.0), (k+c, 1.0), (k+d, 0.0)

The above numeric fuzzy restriction can be graphically explained as elaborated in figure 4.4.

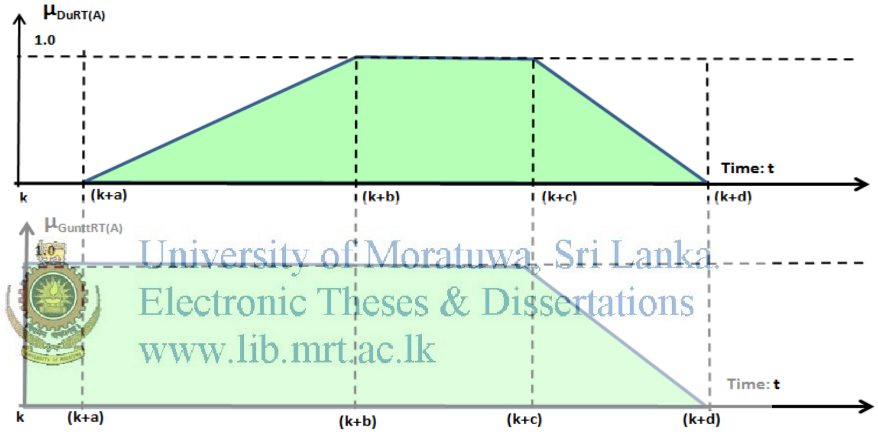


Figure 4.4: Fuzzy Guntt bar analogous to its fuzzy task duration for the typical recurring task

GunttRT(A) can be explained referring to the probability-possibility consistency principle. Therefore $\mu_{DuRT(A)}$ and $\mu_{GunttRT(A)}$ is elaborated with regard to their respective possibility and necessity functions in contrast to each other.

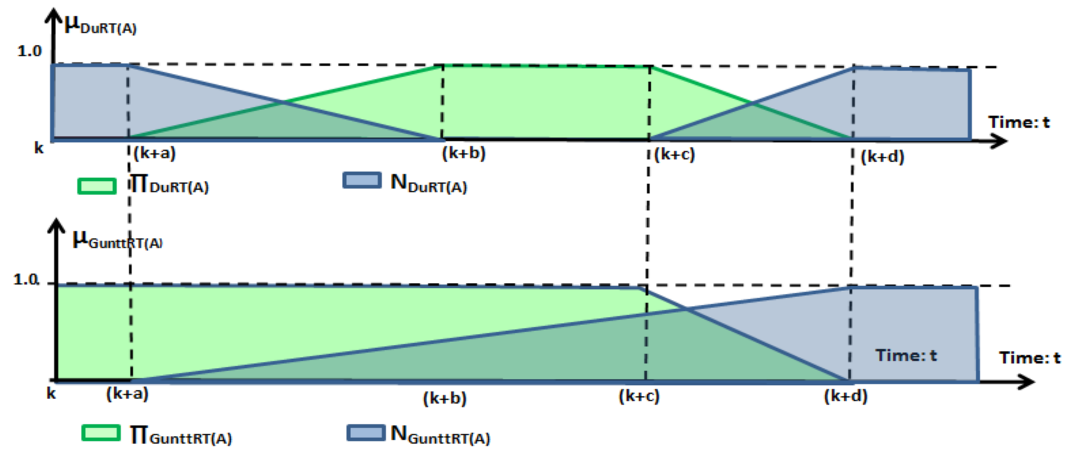


Figure 4.5: Distribution of possibility and necessity functions for the fuzzy duration and Fuzzy Guntt bar for the typical recurring task

$\mu_{DuRT(A)}$ and $\mu_{GunttRT(A)}$ respectively signify the degree of membership for time (t) to meet the physical task completion (i.e. time (t) to become task finishing time) and the degree of membership for the existence of task performance at time (t) (i.e. time (t) to accommodate within the task duration). This is with regard to the timing of one unit of work to be performed under RT(A).

It can be ruled out with figure 4.5, in between the interval (k) to (k+a); $\mu_{GunttRT(A)}$ is to equal one. During the same period $N_{DuRT(A)}$ and $N_{GunttRT(A)}$ shall respectively equal to one and zero whilst $\prod_{DuRT(A)}$ and $\prod_{GunttRT(A)}$ conversely being zero and one to indicate the existence of task performance during the period to be mandatory. During this period the physical task completion is unlikely to happen as in fact the duration of RT(A) then becomes less than its minimum possible time.

During the period (k+a) to (k+c) it is justifiable that $\prod_{DuRT(A)}$ increases and then becomes stable at one. Hence during the period, probability to meet the physical task completion increases along with the time (analogous to figure 4.1). Therefore, probability to exist the task performance is expectable for the period. As a result according to the probability-possibility consistency principle during the period possibility to exist GunttRT(A) at its upper bound of probability is one. Therefore during the period; $\prod_{GunttRT(A)} = 1.0$ and as per the equation: 3.6.1. (6) $\mu_{GunttRT(A)} = 1.0$.

From time (k+a) to (k+c) since $\prod_{DuRT(A)}$ increases to become stable at one $N_{GunttRT(A)}$ is supposedly continue to increase for the period. Because during the period probability of time (t) to meet the physical task completion increases resulting $\prod_{[GunttRT(A)]^c}$ to increase. In this respect as time (t) possesses more degree of membership to become task completion, possibility for the non existence of RT(A)'s performance (i.e. $\prod_{[GunttRT(A)]^c}$) at time (t) continues to increase.

Nevertheless, in contrast to the figure 4.1, in between the time (k+c) to (k+d) probability for RT(A) to meet the physical task completion is expected to decline. This declination appearing in the probability records zero at (k+d) and is not expected to regain with further elapse of time. It is therefore arguable, decrease that records no more upraise in the probability for meeting the physical task completion would result in, the upper bound of probability (i.e. possibility) for the existence of task performance to lower during the same period. Therefore along the downturn of $\prod_{DuRT(A)}$ during the period from (k+c) to (k+d), $\prod_{GunttRT(A)}$ would also decline. This is made necessarily true by the uprising trend of $N_{GunttRT(A)}$ during the period. Therefore, $\mu_{GunttRT(A)}$ presumably decreases for the period.

4.1.3.3 Fuzzy potential for student syndrome (Parkinson law)

This fuzzy variable is adapted to the current study to fulfill the requirement that, safety free optimistic task duration estimate to be realistic. But, according to the theories, along with the rise of assigned task duration and subsequent relaxation, human temptation to expand the performing time to fill the time available tends to increase. This subjective propensity is utilized under the current study as a caliber to plot the permissible fuzzy pessimism in task durations. It is to bring about reconciliation among the generic and pessimistic durations. This is promising to result in fuzzy restriction for the realistic duration estimate.

Nevertheless, potential for student syndrome at any assigned duration to a task is seemingly subjective. This is because this parameter is intangible and notional. The subjective knowledge under student syndrome remarks that it's potential is minimum when the task duration is pragmatically minimum and vise versa. Therefore it is opted to exhibit by a triangular fuzzy set dragging a straight line between the membership function values of zero and one respectively given for the minimum and maximum statistically known task durations.

Potential for student syndrome for the hypothetical recurring task RT(A) for its one unit of work to perform can be denominated as PssRT(A). Its membership function for each statistically observed task duration is then represented by $\mu_{PssRT(A)}$.

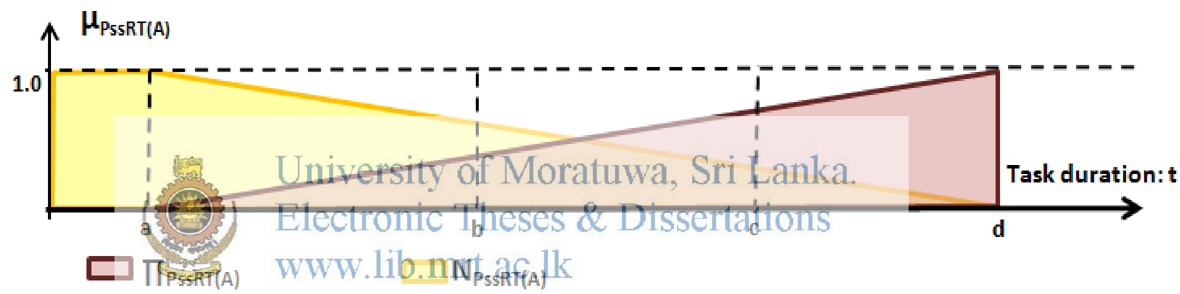


Figure 4.6: Triangular fuzzy set to represent fuzzy potential for student syndrome

$\mu_{PssRT(A)}$ thus represents subjective pessimism ingrained within the statistically observed task duration estimates. Figure 4.6 is its triangular fuzzy illustration. Its numeric fuzzy restrictions are as follows,

TriFN[PssRT(A)]: (a, 0.0), (d, 1.0), (d, 0.0)

4.1.3.4 Fuzzy resources availability

Availability of the specialized crew to perform RT(A) is to be examined in comparison to its resources availability histogram. The recurring phases of the task are to be performed within the contention imposed by the resources availabilities. Resource availability for RT(A) as an antecedent variable is represented in this narration as RaRT(A). Membership function of this resources availability fuzzy set is represented $\mu_{RaRT(A)}$. This is supposedly a trapezoidal fuzzy set due to the fact that it is a fuzzy interval outlined within the resource availability histogram.

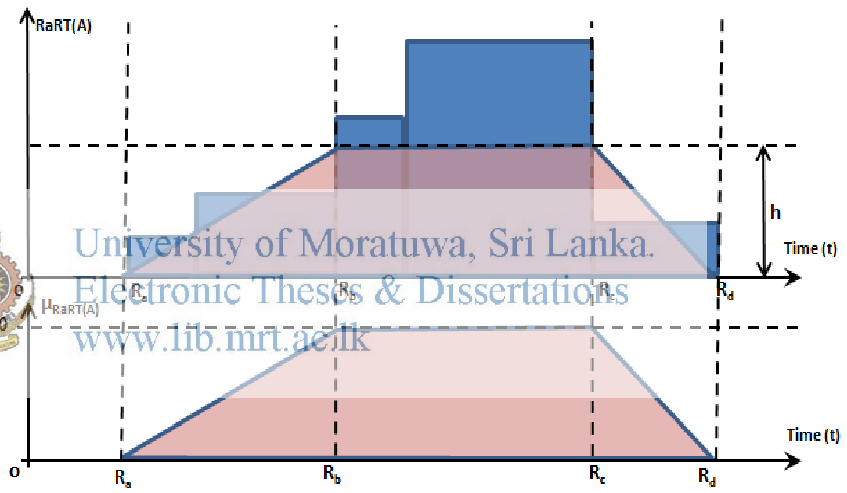


Figure 4.7: Trapezoidal fuzzy set representing the fuzzy resource availability

Starting and finishing dates of the resources availability presents for the interval (R_a)-(R_d) of the trapezoid indicating the longest resource availability period at the lowest resource level (i.e. number of specialized crews available). The fuzzy interval (R_b)-(R_c) is derived at the considered level of resource availability by determining its interval between the external borders of the most outward columns of the histogram. At this determination the area acquired by the trapezoid shall equal to the total area belonging to the histogram.

The trapezoidal representation for the resource availability is then to be transformed in to a trapezoidal fuzzy set. This is to be done by assigning

$\mu_{RaRT(A)}$=1.0 for the fuzzy interval (R_b)-(R_c) of where de facto maximum resource availability is supposed.

Trapezoidal fuzzy numbers imposing fuzzy restriction on RaRT(A) is therefore,

TraFN[RaRT(A)]:(R_a, 0.0), (R_b, 1.0), (R_c, 1.0), (R_d, 0.0)

4.1.3.5 Fuzzy balance project duration

Balance project duration which invariably depletes over the project duration is contemplated as another fuzzy antecedent. This parameter is incorporated in order to enable the proposed approach to exploit the timing of outstanding works at any specific time in between the project starting and finishing times. Fuzzy balance project duration further corresponds to the subjective knowledge that it is in a linear declination trend from the project starting to finishing times. Hence corresponding membership functions ($\mu_{BpdRT(A)}$) would inevitably value for one and zero at starting and finishing milestones of the project.

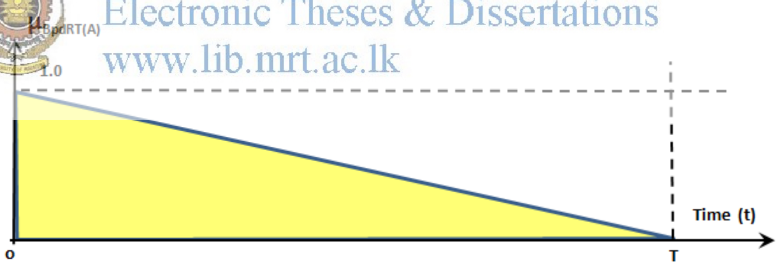


Figure 4.8: Triangular fuzzy set representing fuzzy balance project duration

According to figure 4.8 project duration is represented by (T). With regard to the foregone subjective contention at time; T, $\mu_{BpdRT(A)}$ is zero whilst at time; 0.0 $\mu_{BpdRT(A)}$ is one.

Numeric representation for the above triangular restriction is as follows.

TriFN[BpdRT(A)]: (0, 0.0), (0, 1.0), (T, 0.0)

4.1.4 Step 04: Fuzzy intersections between antecedents

Following the configuration of basic fuzzy sets for articulating further steps of the proposed approach, it is to be discussed the principle operations between them and setting up of their implication method to compose the inferring system.

4.1.4.1 Fuzzy intersection between task duration and potential for student syndrome

The intersection between fuzzy task duration and fuzzy potential for student syndrome interprets for a given task duration the extent of pessimism ingrained in the duration estimate. This is the intersection between the fuzzy generic task duration and the fuzzy potential for student syndrome where in fact the former being an objective and the later being a subjective variable. Potential for student syndrome is scaling up towards the increasing direction of the task duration distribution. Nevertheless, fuzzy intersection (Equation: 3.6.3. (5)) generating the minimum of membership values pertaining to either of the antecedents at any given duration imposes more restriction on the possibility of gaining both the features together. Therefore fuzzy intersection between these two antecedents provides its membership function value at any task duration to be a generic and as well a pessimistic estimate. Hence fuzzy intersection between task duration estimate and potential for student syndrome provides the necessary caliber to curb the safety exempted task duration to be realistic.

Assuming work to be performed under RT(A) as (w) according to Equation: 3.6.3. (6),

TraFN[DuRT(A)]: $(a \otimes w, 0.0), (b \otimes w, 1.0), (c \otimes w, 1.0), (d \otimes w, 0.0)$

TriFN[PssRT(A)]: $(a \otimes w, 0.0), (d \otimes w, 1.0), (d \otimes w, 0.0)$

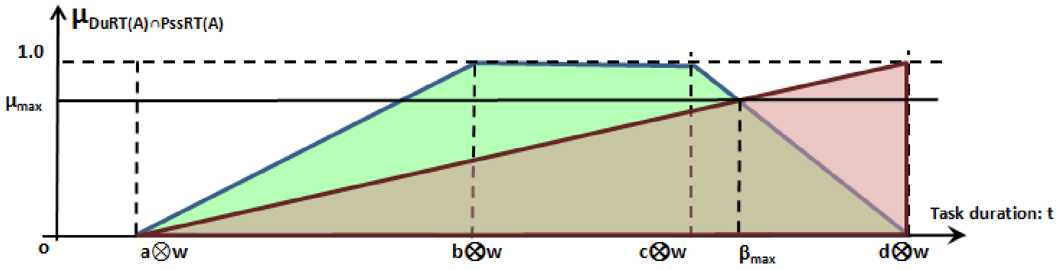


Figure 4.9: Intersection between fuzzy task duration and fuzzy potential for student syndrome

μ_{max} indicates the maximum of the membership value that fuzzy intersection can gain. Therefore, triangular fuzzy restriction forming the intersection between task duration estimate and the task potential for student syndrome [DuRT(A)∩PssRT(A)] can be numerically denominated as below.

TriFN[DuRT(A)∩PssRT(A)]: (a⊗w, 0.0), (β_{max}, μ_{max}), (d⊗w, 0.0)

At this phenomenon DuRT(A) and PssRT(A) exist within the fuzzy interval that all the possibilities for GunttRT(A) remain i.e. between the interval (0-d⊗w).

4.1.4.2 Fuzzy intersection between resource availability and balance project duration

Resource availability and balance project duration are fuzzy sets that exist within the interval of project duration (T). These antecedents correspond to the project related dynamics assumed as fixed as per the drawn limitations to the new approach. Hence fuzzy intersection between resource availability and balance project duration is supposed to be the superset for fuzzy Guntt bar which represents the possibilities for the existence of the task within the interval of project duration.

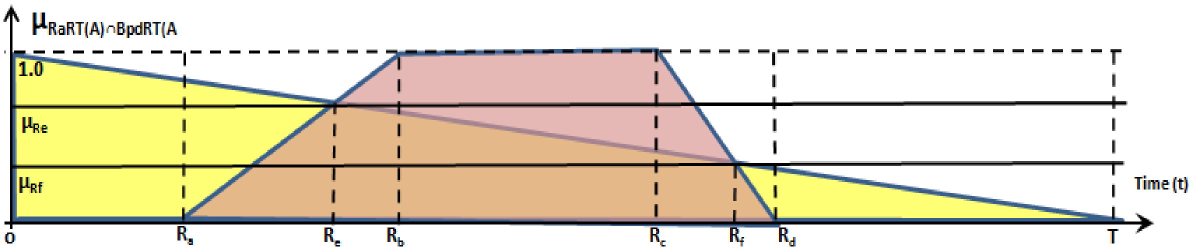


Figure 4.10: Intersection between fuzzy resource availability and fuzzy balance project duration

TraFN[RaRT(A)]: (R_a, 0.0), (R_b, 1.0), (R_c, 1.0), (R_d, 0.0)

TriFN[BpdRT(A)]: (0, 0.0), (0, 1.0), (T, 0.0)

According to Equation: 3.6.3. (5),

FN[RaRT(A)∩BpdRT(A)]: (R_a, 0.0), (R_e, μ_{Re}), (R_f, μ_{Rf}), (R_d, 0.0)

4.1.4.3 Fuzzy intersection between the completion times of two successive critical tasks

Assuming RT(A) and RT(B) as two consecutive recurring tasks positioned at the critical chain of a project schedule it can be explored of their possibilities to gain values on time (t) axis for physical task completions. In this geometrical examination drawing reference to possibility theory it is delineated,

- Possibility and necessity functions of the physical completion times of task RT(B) triggered by its fuzzy task duration, whilst assuming the same functions of RT(A) are constant
- Possibility function of fuzzy Guntt bar of RT(B) as possibility function of RT(A) is stable

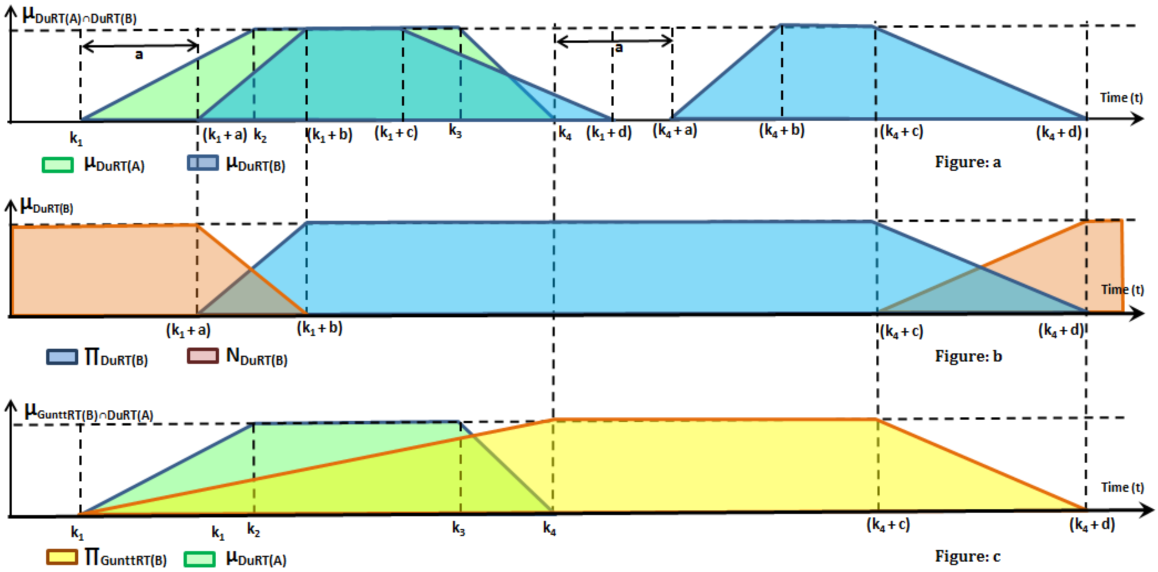


Figure 4.11: Geometrical explanation for the intersection between the fuzzy completion times of two successive critical tasks

According to figure 4.11(a) TraFN[(k_1+a, 0.0), (k_1+b, 1.0), (k_1+c, 1.0), (k_1+d, 0.0)] and TraFN[(k_4+a, 0.0), (k_4+b, 1.0), (k_4+c, 1.0), (k_4+d, 0.0)] respectively denotes the earliest and latest fuzzy physical completion times for RT(B) as fuzzy physical completion times for RT(A) are fixed.

Considering the extreme positions of $\mu_{DuRT(B)}$, figure 4.11(b) explains the $\prod_{DuRT(B)}$ of RT(B). In this explanation all possibilities to position fuzzy completion times of RT(B) are depicted assuming $\prod_{DuRT(A)}$ is constant.

In contrast, figure 4.11(c) submits possibility distribution for the fuzzy Guntt bar of RT(B). Its worthy of signifying that $\prod_{GunttRT(B)}$ intersect RT(A) along with $N_{GunttRT(A)}$ where $\prod_{[GunttRT(A)]^c}$ is expected with regard to the substantiation under section 4.1.3.2.

4.1.5 Step 05: Aggregation of antecedents

In line with the principles outlined under Alpha-cut arithmetic operations, fuzzy completion time of RT(A) assuming the task starting time as (k) can be explained as illustrated in figure 4.12.

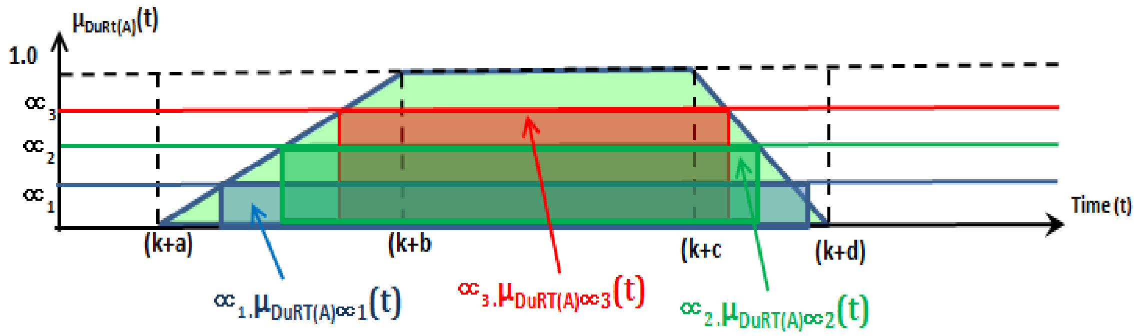


Figure 4.12: Integrating ∝-cuts of DuRT(A) multiplied with their respective ∝ forms the membership function of $\mu_{DuRT(A)}$

Figure 4.12 can be mathematically represented as below.

$$\mu_{DuRT(A)}(t) = \int_{\propto=0}^{\propto=1} [\propto . \mu_{DuRT(A)\propto}(t)]\, d\propto \qquad \text{Equation: 4.1.5. (1)}$$

Therefore the union of all ∝-cuts multiplied with their respective ∝ level would derive the membership function of the fuzzy set. Any ∝-cut at its upper level yields the maximum of membership function for the aggregation of different ∝ levels from zero.

4.1.5.1 Alpha-cut and fuzzy interval arithmetic to obtain the project buffer

The ∝-cut operations blended with fuzzy interval arithmetic operations can be exerted to obtain the optimistic and pessimistic project duration at the determined level of ∝-cut. At this juncture ∝-cut operations are applied to the fuzzy task completion times in order to derive the membership function of the fuzzy Guntt bar pertaining to the task. Minimum and maximum (t) values obtained at the drawn ∝-cut at optimum possibility level is respectively supposed to be the optimistic and pessimistic completion times for the task.

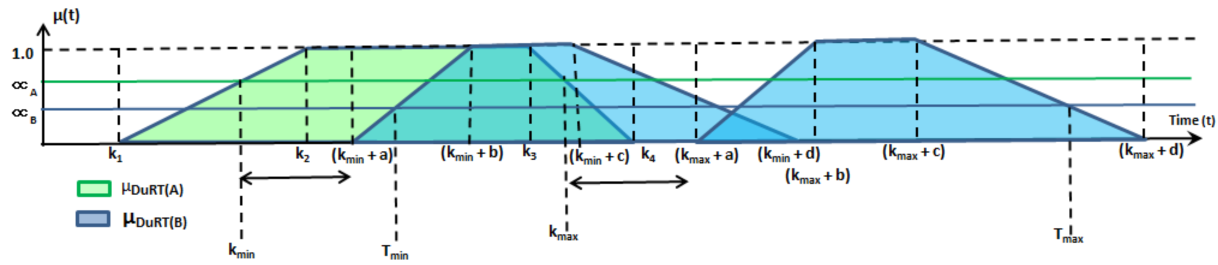


Figure 4.13: Derivation of optimistic and pessimistic scheduling times determined by ∝-cuts

Figure 4.13 delineates the distribution of fuzzy task completion times for RT(B), determined optimistically and pessimistically according to the fixed distribution of $\mu_{DuRT(A)}(t)$ along with predetermined $\propto_A$ level. K_{min} and K_{max} indicate the optimistic and pessimistic completion times for RT(A). Minimum possible duration for RT(B) is supposed to be (a) whilst minimum possible completion time for RT(A) is assumed as K_1. T_{min} and T_{max} renders the optimistic and pessimistic completion times when RT(B) is scheduled to the starting times K_{min} and K_{max} respectively.

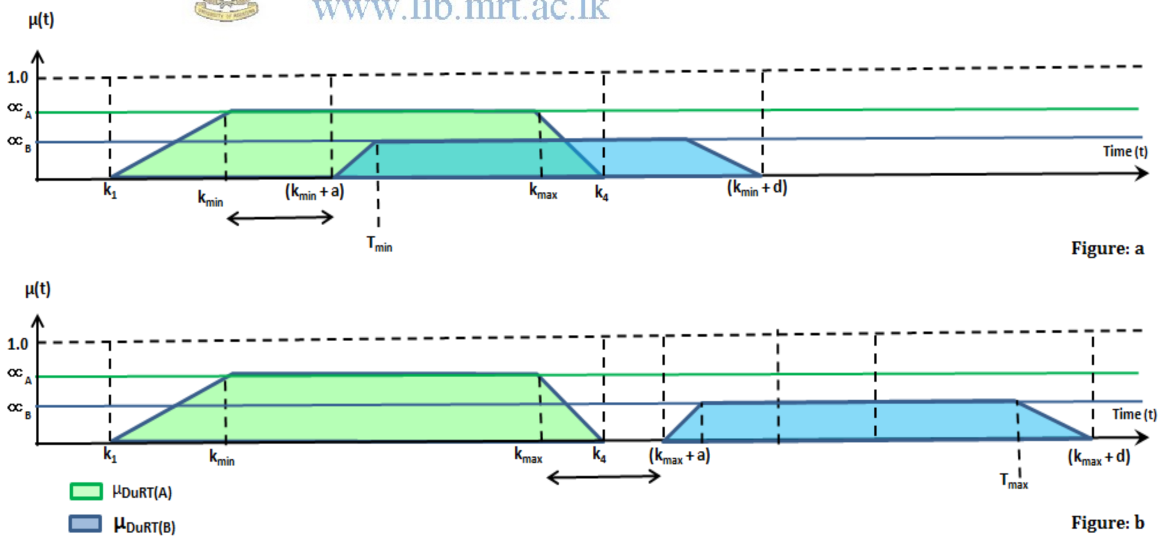


Figure 4.14: Optimistic (Figure: a) and pessimistic (Figure: b) completion times for RT(B) and membership function of DuRT(A) and DuRT(B) at their respective ∝-cuts

Figure 4.14 (a) illustrates $\mu_{DuRT(B)}(t)$ as RT(B) is scheduled to the optimistic starting time K_{min}. Figure 4.14 (b) illustrates $\mu_{DuRT(B)}(t)$ as RT(B) is scheduled to the pessimistic starting time K_{max}. Both the determinations are done respectively at varying $\propto_B$ level of DuRT(B). Theses arithmetical operations are conforming to the equation: 3.6.3. (1).

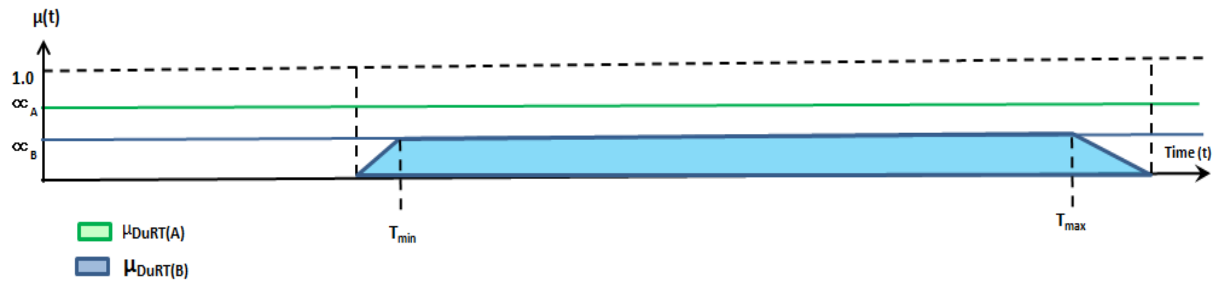


Figure 4.15: Membership function of GunttRT(B) deriving optimistic and pessimistic completion times for RT(B)

Figure 4.15 renders $\mu_{GunttRT(B)}(t)$ taking in to consideration of optimistic and pessimistic schedule of RT(B) and determined level of $\propto_B$. Hence the project schedule fraught of dual recurring tasks RT(A) and RT(B) can be optimistically scheduled for the duration T_{min} whilst T_{max} being its pessimistic schedule duration. Hence the project buffer for this typical dual task project can be interpreted as (T_{max} - T_{min}).

Fuzzy intersection elaborated in figure 4.9 thus convinces the optimistic task duration disperses in between ($a\otimes w$) and ($b\otimes w$), whilst buffer contribution of the task varies between ($a\otimes w$) and ($d\otimes w$). Therefore $\propto_{max}$ renders the maximum of $\propto$ composing more of subsethood of PssRT(A) i.e. pessimism necessitated in scheduling the task, which does not increase as level of $\propto$ increases over $\propto_{max}$. Hence, $\propto_{max}$ establishes the maximum of the level of $\propto$ rendering fuzzy DuRT(A) forming intersection with PssRT(A).

4.1.6 Step 06: Defuzzification based on optimized fuzzy subsethood

The final step of the proposed method specifies the method of determining $\propto$-cut level in order to exert defuzzification. Nevertheless, at this step the fuzzified solution itself is utilized to generate the optimistic and pessimistic scheduling times. The level of ∝-cut to be determined in order to establish the fuzzy Guntt bar of each recurring task at the same time their respective resource availability and balance project duration is optimally saturated in integrity. At this juncture the argument put forward is, the fuzzy intersection between resource availability and balance project duration [i.e. $\mu_{RaRT(A)\cap BpdRT(A)}(t)$] being the superset of fuzzy Guntt bar [$\mu_{GunttRT(A)}(t)$], the later shall be performed within the possibilities enabled by the former.

The degree of subsethood that GunttRT(A) exhibits in terms of RaRT(A)∩BpdRT(A) can be represented as S[RaRT(A)∩BpdRT(A), GunttRT(A)].

$$GunttRT(A) \subseteq RaRT(A) \cap BpdRT(A)$$

$$S[RaRT(A)\cap BpdRT(A), GunttRT(A)] = \frac{M\{[RaRT(A)\cap BpdRT(A)]\cap[GunttRT(A)]\}}{M[RaRT(A)\cap BpdRT(A)]}$$

Equation: 4.1.6. (1)

M represents the cardinality measure of fuzzy sets with reference to the figure 4.10 and 4.5 as below.

$$M[RaRT(A)\cap BpdRT(A)] = \int_{t=0}^{t=T} \mu_{RaRT(A)\cap BpdRT(A)}(t)\, dt$$ Equation: 4.1.6. (2)

$$M[GunttRT(A)] = \int_{t=0}^{t=T} \mu_{GunttRT(A)}(t)\, dt$$ Equation: 4.1.6. (3)

Thus ∝-cut levels of all the recurring tasks are determined minimizing the sum of supersethood i.e. minimizing the sum of violations to subsethood for all the recurring tasks housed in project critical chain. A non-linear optimization technique is to be exerted in order to obtain the mathematical solution to this subsethood optimization problem.

Figure 4.16 illustrates the distribution of membership function of fuzzy Guntt bar of a typical recurring task type RT(A) [$\mu_{GunttRT(A)}(t)$] within the supersethood constructed by the fuzzy intersection between balance project duration [$\mu_{BpdRT(A)}(t)$] and as appropriate for the performance of RT(A), fuzzy resource availability [$\mu_{RaRT(A)}(t)$]. Hence, the fuzzy intersection is denoted by [$\mu_{RaRT(A) \cap BpdRT(A)}(t)$].

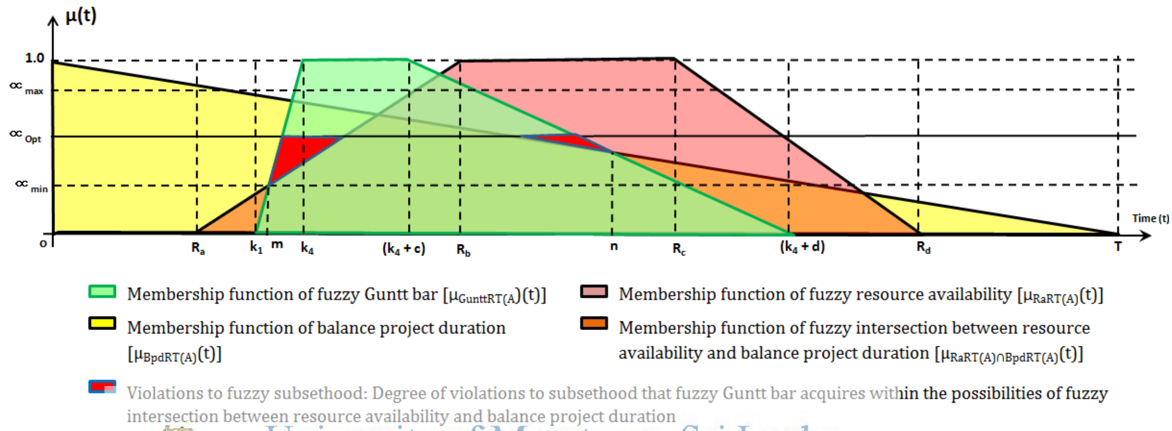


Figure 4.16: Graphical explanation for violations to fuzzy subsethood and constraints established under the optimized solution

$\mu_{GunttRT(A)}(t)$ as delineated in figure 4.16 is obtained in accordance with the drawn contentions under the subsection 4.1.4.3 and the interpretations drawn by the figure 4.11 (c) thereof. Similarly $\mu_{RaRT(A) \cap BpdRT(A)}(t)$ is fabricated conforming to the subsection 4.1.4.2 in contrast with figure 4.10. $\propto_{max}$ is obtained as the highest level of membership value obtainable for the intersection $\mu_{DuRT(A) \cap PssRT(A)}(t)$ as explanations are drawn under the subsection 4.1.4.1. $\propto_{min}$ is the highest level of $\propto$ enabling $\mu_{GunttRT(A)}(t)$ to form zero subsethood with $\mu_{RaRT(A) \cap BpdRT(A)}(t)$. $\propto_{Opt}$ denotes the level of $\propto$ derived for the task but appropriate with the optimized solution.

The optimization of fuzzy subsethood is resulted by the minimization of violations to fuzzy subsethood. Therefore, the non-linear optimization problem yields minimization problem of violations to fuzzy subsethood. It is simultaneously assessed along with the non-linear optimization problem; the optimistic and pessimistic scheduling durations for all recurring tasks (determined by respective $\propto$-levels) whilst their membership functions of fuzzy Guntt bars form minimum aggregated violations to their supersets. The established constraints, decision variables and the objective function with regard to the non-linear optimization problem are as follows.

The optimistic starting time of the task (determined by fuzzy Guntt bar) is under the constraint of being on par with or later than the earliest day of relevant resource availability.

Hence,

$$K_1 \geq R_a \qquad \text{Equation: 4.1.6. (4)}$$

Conversely to the above pessimistic completion time of the task (determined by fuzzy Guntt bar) is on or before the latest day of resource availability.

Hence,

$$(K_4+d) \leq R_d \qquad \text{Equation: 4.1.6. (5)}$$

The level of $\propto$ resulted by the optimized solution i.e. $\propto_{Opt}$ is to obtain a value in between $\propto_{min}$ and $\propto_{max}$.

Hence,

$$\propto_{Opt} \geq \propto_{min} \qquad \text{Equation: 4.1.6. (6)}$$

And,

$$\propto_{Opt} \leq \propto_{max} \qquad \text{Equation: 4.1.6. (7)}$$

$\propto_{Opt}$ is established as the decision variable of the task under consideration. Depending on the determined level of $\propto$ within the above boundaries, the degree of violations to fuzzy subsethood measure varies. Hence in the optimization mechanism, $\propto_{Opt}$ is determined against each recurring task on critical chain as sum of violations to aforesaid subsethood become minimal.

The objective function to be minimized is interpreted as below.

Violation to fuzzy subsethood =

$$\int_{t=m}^{t=n} \mu_{GunttRT(A)\propto Opt}(t)\, dt - \int_{t=m}^{t=n} \mu_{[RaRT(A)\cap BpdRT(A)]\propto Opt}(t)\, dt$$

4.2 Summary

The chapter presents the underlying six steps as proposed by the new buffer sizing approach.

1. Construction of CDF of the probabilities of a typical recurring task's duration distribution
2. Identification method for the project critical chain
3. Fuzzification of the following antecedents in relation to a typical recurring task,

(a) Duration distribution

(b) Guntt bar distribution

(c) Distribution of Potential for student syndrome (Parkinson law)

(d) Distribution of Resource availability

(e) Distribution of Balance project duration

4. Identifying the necessary fuzzy operators and drawing up of implication method centering typical recurring tasks

(a) Identification of the necessary fuzzy operators

(b) Creating fuzzy intersection between task duration and potential for student syndrome

(c) Creating fuzzy intersection between resource availability and balance project duration

(d) Creating fuzzy intersection between completion times of two successive tasks

5. Implementing α-cut and fuzzy interval arithmetic in order to obtain affordable solutions for the project buffer

6. Defuzzification based on optimized fuzzy subsethood acquired by fuzzy Guntt bar within the supersethood drawn by fuzzy intersection between balance project duration and resource availability fuzzy sets

CHAPTER 05

EMPIRICAL TEST ON VALIDATING THE PROPOSED APPROACH

5.1 Introduction

The proposed buffer sizing approach delineated in the presiding chapter is examined for its practicality in water distribution projects in order to ensure its validation.

A specific zone of a water distribution project where partial completion is imposed by the project formalities via a milestone completion date is explored. The selected water distribution project is below unfolded in relation to its focal zone of construction for the implementation of proposed approach under the case study.

Table 5.1: Summary of project particulars in relation to the selected zone of construction for the case study

Location	Colombo district, Western Province, Sri Lanka
Duration for the zone completion	210 days
Stipulated contract sum for the works executed under the zone	SL Rs. 500, 000, 000.00
Length of uPVC laying (90mm – 225mm diameter)	9, 800m
Length of DI laying (250mm-350mm diameter)	1, 860m
Number of valve chambers to be constructed	17 Numbers
Length of road identified to be reinstated	5, 500m

The identified zone of partial completion under the selected water distribution project is denominated as the case study project. Appendix 01 renders the schedule and the work break down structure for the case study project.

5.2 Aggregation of duration statistics

Along with the recurring tasks of a water distribution project as denominated in proposal, duration statistics for the unit of work performance are identified. Scrutinizing a sample of four (04) independent project schedules complying the criteria specified in section 3.1, the underlying number of unbiased sample observations against each recurring task were found. It is signified that each observation under each recurring task refers to a unique work condition at identical locations within the project boundaries. Nevertheless conforming to the established principles under CCPM the current study does not probe in to the project related characteristics of the sample observations. Hence the study only accumulates individual duration statistics for the mere prospective of constructing duration distribution in relation to the unit of work under each recurring task.

Table 5.2: Number of unbiased observations for the identified unit of work performance under each recurring task

Recurring task	Unit of work	Number of observations for task duration				
		Project 01	Project 02	Project 03	Project 04	Total
uPVC Laying	1.0 Km	12	13	9	9	43
DI Laying	1.0 Km	8	10	7	5	30
Culvert crossing	1.0 Nr	11	12	7	8	38
Chamber construction	1.0 Nr	9	11	8	7	35
Pressure testing	1.0 Km	6	12	11	12	41
Flushing and disinfection	1.0 Km	10	11	8	12	41
Reinstatement	1.0 Km	8	13	8	8	37

5.3 Step 01: Construction of CDF of probabilities of task durations

The duration values observed against the recurring tasks are utilized in order to derive the CDF of probabilities for the task duration distribution for its one unit of work. Independent observations obtained for each recurring task are then categorized in to equal twenty (20) number of groups divided equally among the largest and the smallest observations. Frequency obtained against each group of observation is then divided by the total number of observations to derive the CDF value.

Following the above analysis at 95% confidence interval is arrived. Confining the computations to this 95% confidence interval trapezoidal or triangular representation for the CDF of per unit task duration distribution is obtained for each recurring task. Table 5.3 illustrates CDF of per unit of work task duration distribution and their trapezoidal or triangular representation at 95% confidence interval.

Table 5.3: Trapezoidal and triangular representation for the CDF of probabilities of task duration distribution at 95% confidence

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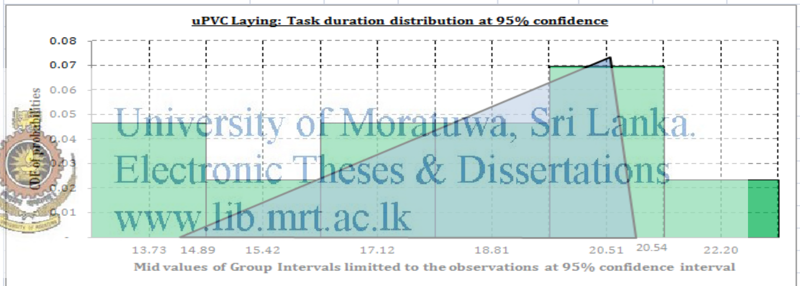
<tr><td colspan="2">

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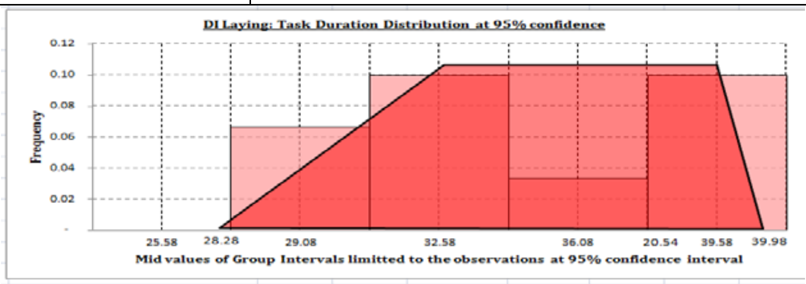
<tr><td>uPVC Laying</td><td>TraNr at duration axis {14.89, 20.51, 20.51, 20.54}</td></tr>
<tr><td colspan="2">
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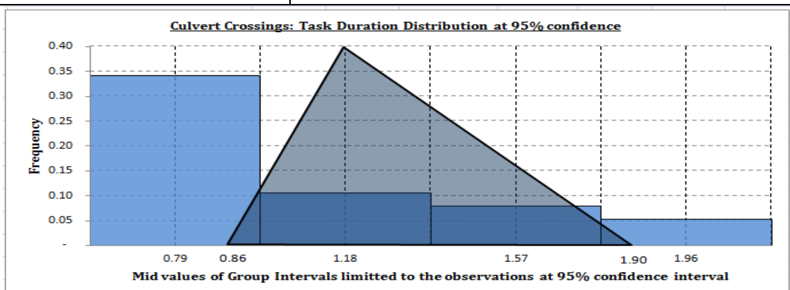
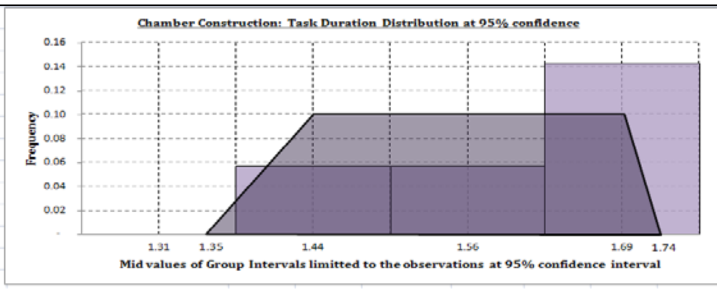
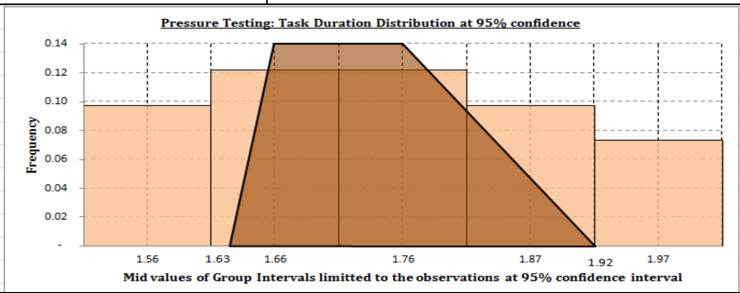
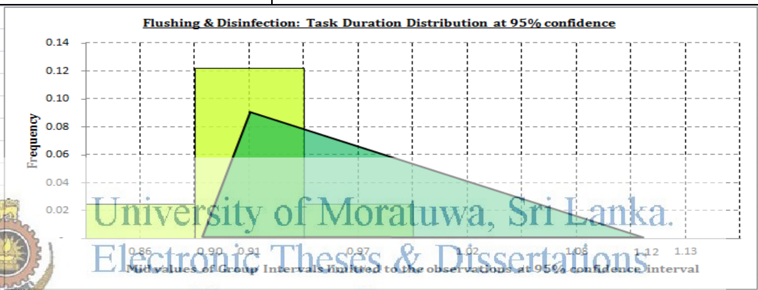
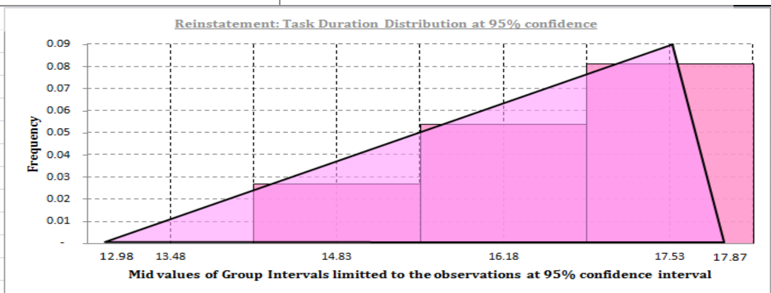
<tr><td>DI Laying</td><td>TraNr at duration axis {28.28, 32.58, 39.58, 39.98}</td></tr>
<tr><td colspan="2">
</td></tr>
<tr><td>Culvert crossing</td><td>TraNr at duration axis {0.86, 1.18, 1.18, 1.90}</td></tr>
</table>

	
Chamber construction	TraNr at duration axis {1.35, 1.44, 1.69, 1.74}
	
Pressure testing	TraNr at duration axis {1.63, 1.66, 1.76, 1.92}
	
Flushing and disinfection	TraNr at duration axis {0.90, 0.91, 0.91, 1.12}
	
Reinstatement	TraNr at duration axis {12.98, 17.53, 17.53, 17.87}

5.4 Step 02: Identification of the critical chain of the case study project

At the same time the tasks are arrayed according to their dependency relationships, different chains of dependant events are identified (Appendix 01). Tasks are then assigned the shortest possible durations at 95% confidence as determined under step 01. The longest chain of dependant tasks devoid of any slack time is identified as the critical chain. Resource leveling is not centered as a matter of concern due to the unavailability of the fact; whether the selected zone construction under the principal project schedule is leveled for resources.

Table 5.4: Critical chain determined for the case study project

Task ID	RT(#)	RECURRING TASK	PATH CATEGORY	Unit	Work	INITIAL DURATION (Days)	PREDICESSOR ACTIVITY	START TIME (Day)	FINISH TIME (Day)
3	RT(01)	uPVC Laying	Critical Path	Km	0.90	20.00	START	1	20
4	RT(02)	DI Laying	Critical Path	Km	0.75	25.00	3	21	45
5	RT(03)	Culvert Crossing	Critical Path	Nr	4.00	5.00	4	46	50
19	RT(04)	uPVC Laying	Critical Path	Km	2.10	45.00	5	51	95
20	RT(05)	DI Laying	Critical Path	Km	0.60	27.00	19	96	122
21	RT(06)	Culvert Crossing	Critical Path	Nr	8.00	12.00	20	123	134
22	RT(07)	Chamber Construction	Critical Path	Nr	5.00	8.00	14,21	135	142
23	RT(08)	Pressure Testing	Critical Path	Km	2.20	4.00	16,22	143	146
24	RT(09)	Flushing and Disinfection	Critical Path	Km	2.20	2.00	23	147	148
29	RT(10)	Pressure Testing	Critical Path	Km	1.10	2.00	24,28	149	150
30	RT(11)	Flushing and Disinfection	Critical Path	Km	1.10	1.00	29	151	151
31	RT(12)	Reinstatement	Critical Path	Km	0.60	10.00	17,30	152	161
39	RT(13)	Reinstatement	Critical Path	Km	0.40	6.00	31,38	162	167
46	RT(14)	Reinstatement	Critical Path	Km	1.60	25.00	39,45	168	192
53	RT(15)	Reinstatement	Critical Path	Km	0.50	8.00	46,52	193	200
60	RT(16)	Reinstatement	Critical Path	Km	0.50	10.00	53,59	201	210

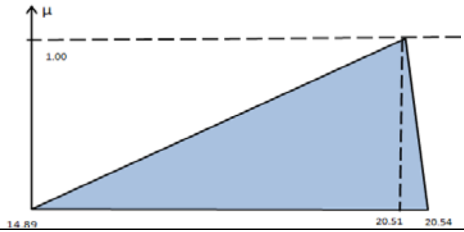
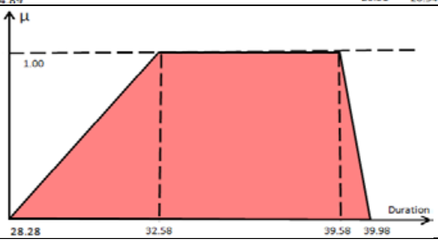
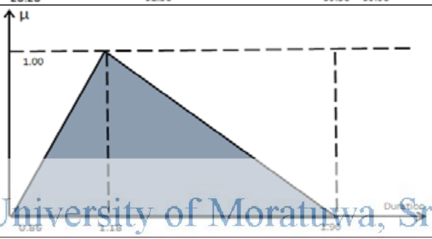
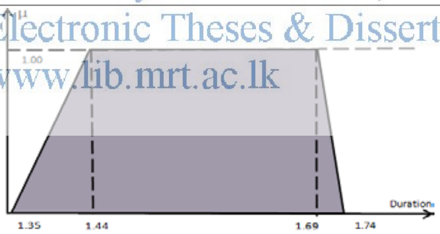
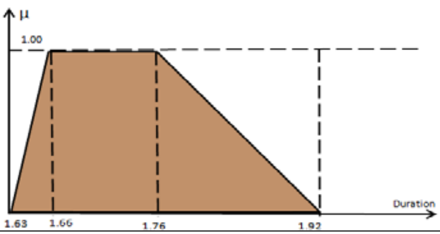
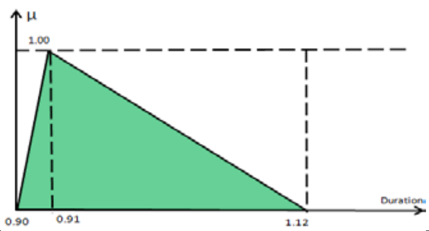
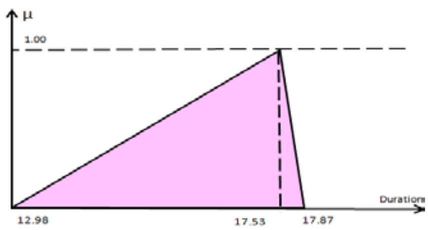
5.5 Step 03: Fuzzification of antecedents to the case study

In line with the principles of possibility theory as set out under the research methodology, antecedents of the case study project are fuzzified.

5.5.1 Fuzzy task duration distributions

CDF of probabilities at 95% confidence interval for the duration distribution of each recurring task is utilized to generate the fuzzy duration distributions. The duration value possessing the highest CDF is given the degree of membership of one whilst the membership degrees of outstanding possible durations are scaled between zero and one. The results are exhibited in table 5.5.

Table 5.5: Fuzzy restrictions obtained for the duration distributions pertaining to the recurring tasks under case study

Recurring Task	Fuzzy Restriction	Fuzzy Numbers
uPVC Laying		Trapezoidal fuzzy numbers: {(14.89, 0.00), (20.51, 1.00), (20.51, 1.00), (20.54, 0.00)
DI Laying		Trapezoidal fuzzy numbers: {(28.28, 0.00), (32.58, 1.00), (39.58, 1.00), (39.98, 0.00)
Culvert crossing		Trapezoidal fuzzy numbers: {(0.86, 0.00), (1.18, 1.00), (1.18, 1.00), (1.90, 0.00)
Chamber construction		Trapezoidal fuzzy numbers: {(1.35, 0.00), (1.44, 1.00), (1.69, 1.00), (1.74, 0.00)
Pressure testing		Trapezoidal fuzzy numbers: {(1.63, 0.00), (1.66, 1.00), (1.76, 1.00), (1.92, 0.00)
Flushing and disinfection		Trapezoidal fuzzy numbers: {(1.90, 0.00), (0.91, 1.00), (0.91, 1.00), (1.12, 0.00)
Reinstatement		Trapezoidal fuzzy numbers: {(12.98, 0.00), (17.53, 1.00), (17.53, 1.00), (17.87, 0.00)

5.5.2 Construction of fuzzy Guntt bar

Guntt bars that represent the task performance between the starting and finishing times of recurring tasks forming the critical chain of case study project schedule are fuzzified. Fuzzy Guntt bars are thus formed taking in to account of the most optimistic and pessimistic distributions for the task completion times to model possibility function ($\prod_{GunttRT(A)}$) of the existence of task performance. The necessity function ($N_{GunttRT(A)}$) representing the complement of the former $[GunttRT(A)]^c$ outlines the dispersion of fuzzy Guntt bars corresponding to the presiding and succeeding tasks.

Table 5.6 presents the numeric results obtained for the optimistic and pessimistic completion times corresponding for each recurring task within the critical chain of the case study project. Fuzzy Guntt bars are thus configured utilizing the numeric results for optimistic and pessimistic task completions along with the necessity functions of adjoining tasks. The delineation of the results according to the figure 5.1 is analogous to the contention drawn under proposal for fuzzy intersection between the completion times of two successive critical tasks.

The results exhibit the most optimistic schedule with no safety, i.e. buffer reserved within the task duration being 154 days, whilst the most pessimistic duration being 221 days containing maximum of safety reservation.

Table 5.6: Fuzzy restrictions for the optimistic and pessimistic dispersion of fuzzy task completion times

Task ID	RT(#)	RECURRING TASK	Unit	Work	PREDICESSOR ACTIVITY	Optimistic ScheduleTrapezoidal fuzzy numbers								Pessimistic ScheduleTrapezoidal fuzzy numbers							
						$(K_1+a\otimes w)$	$\mu_{RT(\#)}$	$(K_1+b\otimes w)$	$\mu_{RT(\#)}$	$(K_1+c\otimes w)$	$\mu_{RT(\#)}$	$(K_1+d\otimes w)$	$\mu_{RT(\#)}$	$(K_4+a\otimes w)$	$\mu_{RT(\#)}$	$(K_4+b\otimes w)$	$\mu_{RT(\#)}$	$(K_4+c\otimes w)$	$\mu_{RT(\#)}$	$(K_4+d\otimes w)$	$\mu_{RT(\#)}$
3	RT(01)	uPVC Laying	Km	0.90	START	13.00	-	18.00	1.00	18.00	1.00	18.00	-	13.00	-	18.00	1.00	18.00	1.00	18.00	-
4	RT(02)	DI Laying	Km	0.75	3	34.00	-	37.00	1.00	43.00	1.00	43.00	-	39.00	-	42.00	1.00	48.00	1.00	48.00	-
5	RT(03)	Culvert Crossing	Nr	4.00	4	37.00	-	39.00	1.00	39.00	1.00	42.00	-	51.00	-	53.00	1.00	53.00	1.00	56.00	-
19	RT(04)	uPVC Laying	Km	2.10	5	68.00	-	80.00	1.00	80.00	1.00	80.00	-	87.00	-	99.00	1.00	99.00	1.00	99.00	-
20	RT(05)	DI Laying	Km	0.60	19	85.00	-	88.00	1.00	92.00	1.00	92.00	-	116.00	-	119.00	1.00	123.00	1.00	123.00	-
21	RT(06)	Culvert Crossing	Nr	8.00	20	92.00	-	94.00	1.00	94.00	1.00	100.00	-	130.00	-	132.00	1.00	132.00	1.00	138.00	-
22	RT(07)	Chamber Construction	Nr	5.00	14,21	99.00	-	99.00	1.00	100.00	1.00	101.00	-	145.00	-	145.00	1.00	146.00	1.00	147.00	-
23	RT(08)	Pressure Testing	Km	2.20	16,22	103.00	-	103.00	1.00	103.00	1.00	103.00	-	151.00	-	151.00	1.00	151.00	1.00	151.00	-
24	RT(09)	Flusing and Disinfection	Km	2.20	23	105.00	-	105.00	1.00	105.00	1.00	105.00	-	153.00	-	153.00	1.00	153.00	1.00	153.00	-
29	RT(10)	Pressure Testing	Km	1.10	24,28	107.00	-	107.00	1.00	107.00	1.00	107.00	-	155.00	-	155.00	1.00	155.00	1.00	155.00	-
30	RT(11)	Flusing and Disinfection	Km	1.10	29	108.00	-	108.00	1.00	108.00	1.00	108.00	-	156.00	-	156.00	1.00	156.00	1.00	156.00	-
31	RT(12)	Reinstatement	Km	0.60	17,30	116.00	-	119.00	1.00	119.00	1.00	119.00	-	164.00	-	167.00	1.00	167.00	1.00	167.00	-
39	RT(13)	Reinstatement	Km	0.40	31,38	121.00	-	123.00	1.00	123.00	1.00	123.00	-	172.00	-	174.00	1.00	174.00	1.00	174.00	-
46	RT(14)	Reinstatement	Km	1.60	39,45	142.00	-	149.00	1.00	149.00	1.00	150.00	-	195.00	-	202.00	1.00	202.00	1.00	203.00	-
53	RT(15)	Reinstatement	Km	0.50	46,52	148.00	-	151.00	1.00	151.00	1.00	151.00	-	209.00	-	212.00	1.00	212.00	1.00	212.00	-
60	RT(16)	Reinstatement	Km	0.50	53,59	154.00	-	157.00	1.00	157.00	1.00	157.00	-	218.00	-	221.00	1.00	221.00	1.00	221.00	-

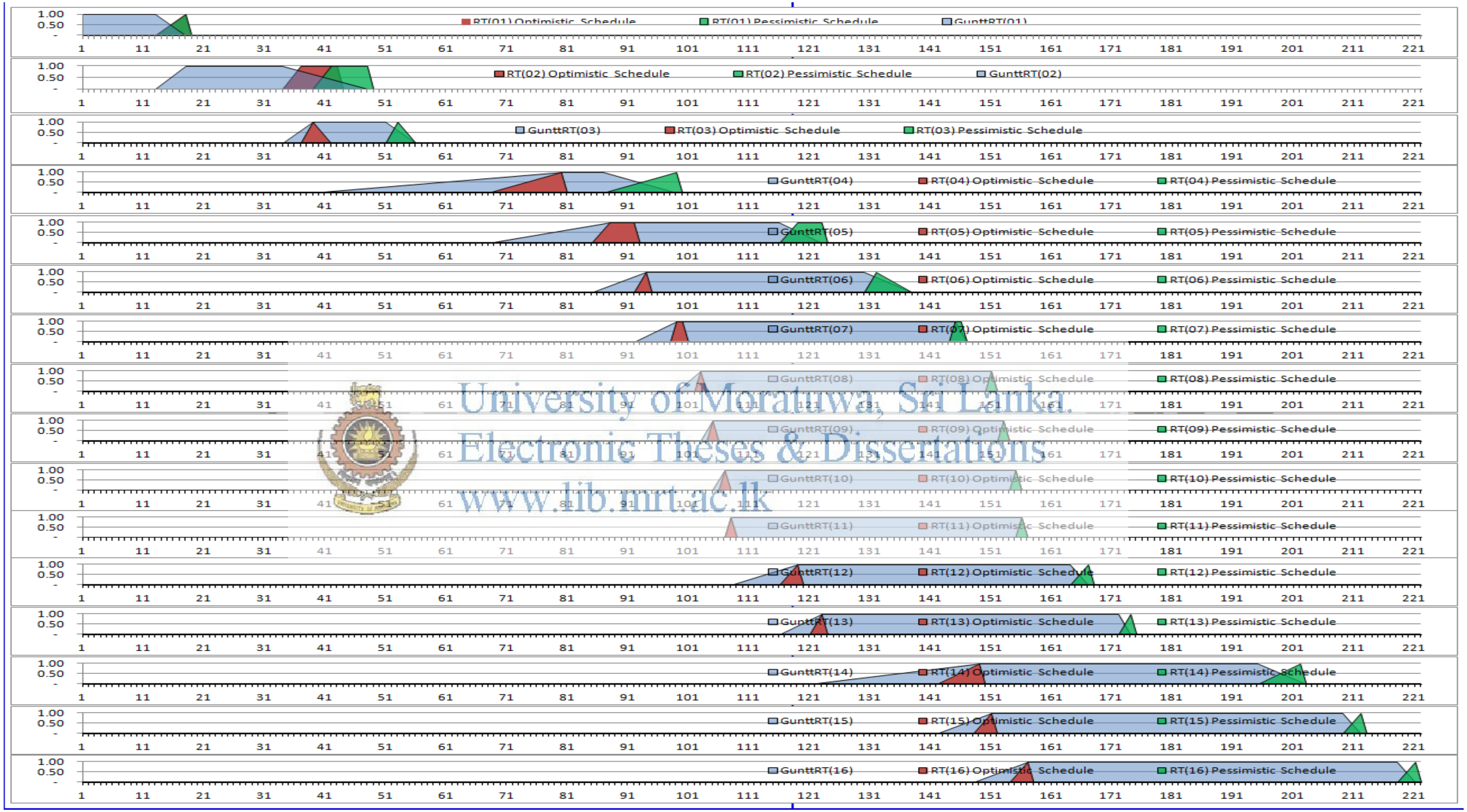
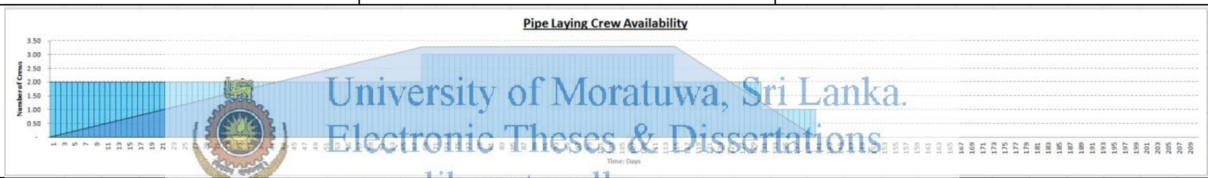
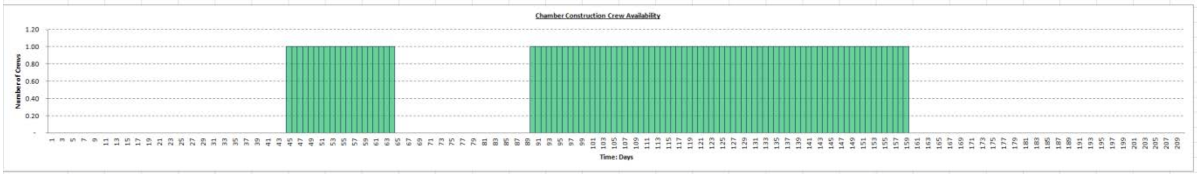


Figure 5.1: Distribution of fuzzy optimistic and pessimistic task completion times and corresponding fuzzy Guntt bars

5.5.3 Fuzzy resources availability of the case study

The categories of resources i.e. crews, planned to be deployed under the case study project are identified against their relevance to the recurring tasks. Histograms indicating the resource availabilities via the number of crews available along the time line are then utilized to construct the fuzzy sets for resource availabilities as argued in research methodology. Table 5.7 depicts the formation of fuzzy sets for resource availabilities in line with their appropriate recurring tasks being utilized.

Table 5.7: Fuzzy representation for resource availabilities of the case study

Resource (Crew) type	**Task usage**	**Trapezoidal fuzzy numbers for resources availability**
Pipe laying	(a) uPVC Laying (b) DI Laying (c) Culvert crossing	{(0.00, 0.00), (69.00, 1.00), (114.00, 1.00), (140, 0.00)}
 		
Chamber construction	(a) Chamber construction	{(90.00, 0.00), (90.00, 1.00), (159.00, 1.00) (159.00, 0.00)} Resource availability not supplementing for the critical chain between days 45 to 64 ignored from calculations
 		
Testing and commissioning	(a) Pressure testing (b) Flushing and disinfection	{(55.00, 0.00), (130.00, 1.00), (145.00, 1.00) (165.00, 0.00)}
 		

Reinstatement	(a) Reinstatement	{(0.00, 100.00), (160.00, 1.00), (195.00, 1.00), (210, 0.00)} Resource availability not supplementing for the critical chain between days 55 to 85 ignored from calculations
		

5.6 Step 04: Exerting fuzzy operators and the implication method for the case study

Conforming to the developed contentions in the proposal the underlying fuzzy intersections are formed in between fuzzy antecedents.

5.6.1 Fuzzy intersection between task duration estimate and potential for student syndrome under the case study

Fuzzy task duration estimates for the recurring tasks which are objectively developed under step 03 of the cast study are then examined for their fuzzy intersection with the subjective potential for student syndrome. Table 5.8 illustrates the observances relevant for this examination. Thus obtained $\propto_{max}$ denotes the maximum of $\propto$ levels which forms fuzzy sets for the task durations composed of pessimisms within their estimates.

5.6.2 Fuzzy intersection between resource availability and balance project duration of the case study

Fuzzy resource availabilities constructed under step 03 are examined for their intersection with balance project duration. Table 5.9 depicts the results.

Table 5.8: Fuzzy intersection between task duration estimates and potential for student syndrome

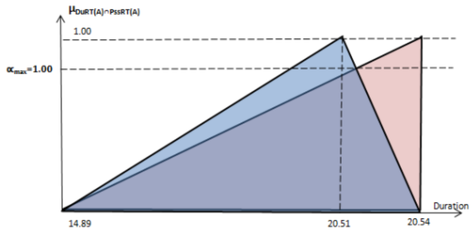
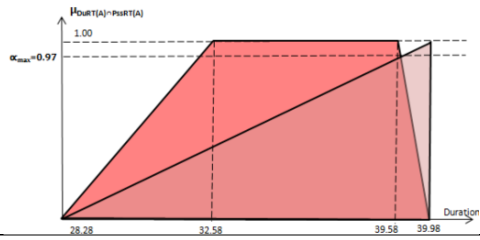
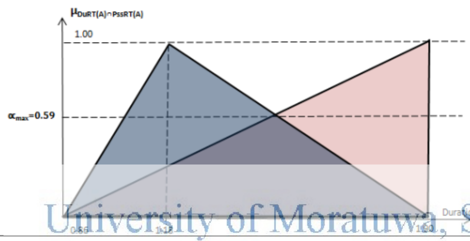
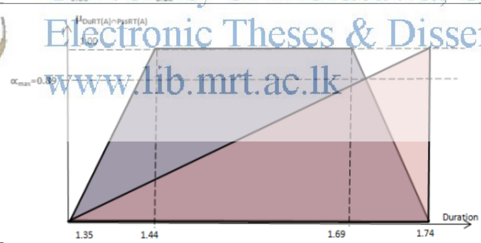
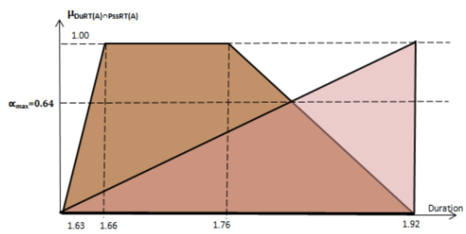
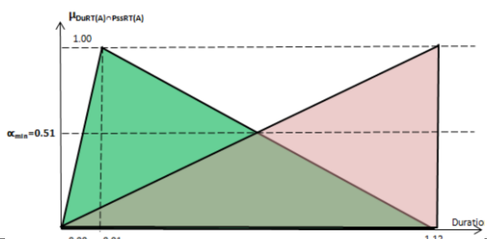
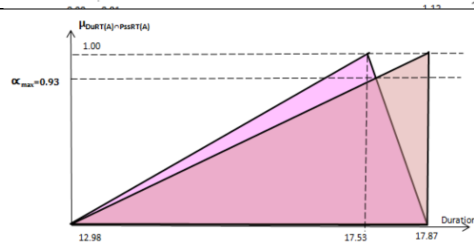
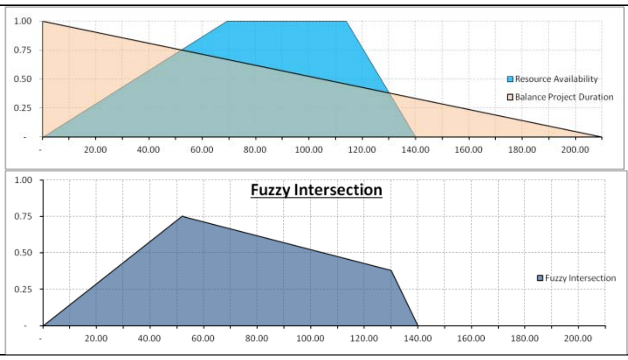
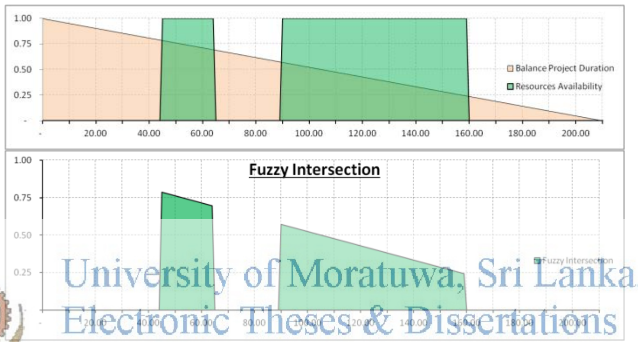
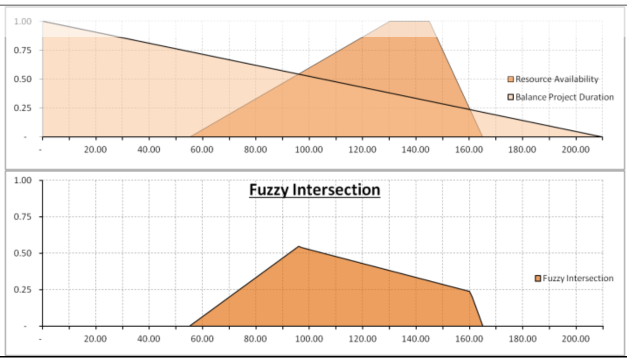
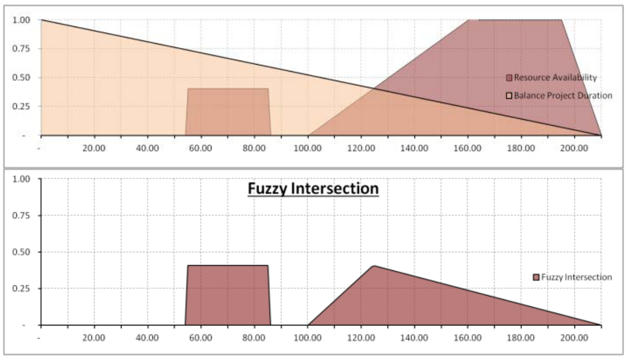
Recurring Task	Fuzzy intersection: DuRT(A)∩PssRT(A)	Fuzzy Numbers
uPVC Laying		Trapezoidal fuzzy numbers: {(14.89, 0.00), (20.51, 1.00), (20.51, 1.00), (20.54, 0.00)} $\propto_{max}$=1.00
DI Laying		Trapezoidal fuzzy numbers: {(28.28, 0.00), (39.63, 0.97), (39.63, 0.97), (39.98, 0.00)} $\propto_{max}$=0.97
Culvert crossing		Trapezoidal fuzzy numbers: {(0.86, 0.00), (1.47, 0.59), (1.47, 0.59), (1.90, 0.00)} $\propto_{max}$=0.59
Chamber construction		Trapezoidal fuzzy numbers: {(1.35, 0.00), (1.70, 0.89), (1.70, 0.89), (1.74, 0.00)} $\propto_{max}$=0.89
Pressure testing		Trapezoidal fuzzy numbers: {(1.63, 0.00), (1.82, 0.64), (1.82, 0.64), (1.92, 0.00)} $\propto_{max}$=0.64
Flushing and disinfection		Trapezoidal fuzzy numbers: {(1.90, 0.00), (1.01, 0.59), (1.01, 0.59), (1.12, 0.00)} $\propto_{max}$=0.59
Reinstatement		Trapezoidal fuzzy numbers: {(12.98, 0.00), (17.53, 0.93), (17.53, 0.93), (17.87, 0.00)} $\propto_{max}$=0.93

Table 5.9: Fuzzy intersection between resource availability and balance project duration

Resource (Crew) type	Fuzzy intersection: RaRT(A)∩BpdRT(A)	Fuzzy numbers
Pipe laying		Trapezoidal fuzzy numbers: {(0.00, 0.00), (52.00, 0.75), (130.00, 0.38), (140, 0.00)}
Chamber construction		Trapezoidal fuzzy numbers: {(90.00, 0.00), (90.00, 0.57), (159.00, 0.24), (159, 0.00)} Resource availability not supplementing for the critical chain between days 45 to 64 ignored from calculations
Testing and commissioning		Trapezoidal fuzzy numbers: {(55.00, 0.00), (96.00, 0.54), (160.00, 0.24), (165, 0.00)}
Reinstatement		Trapezoidal fuzzy numbers: {(100.00, 0.00), (124.00, 0.40), (124.00, 0.40), (210, 0.00)} Resource availability not supplementing for the critical chain between days 55 to 85 ignored from calculations

5.7 Step 05: Exertion of Alpha-cut arithmetic and aggregation method to the case study

∝-cut arithmetic operations are exerted to the membership functions of respective task completion times of the recurring tasks in critical chain under the case study project. Similarly as contended under research methodology ∝-cut arithmetic operations derive optimistic and pessimistic task completion times. Thus possibility distribution for fuzzy Guntt bar is obtained taking in to account of necessity distribution pertaining to the preceding task under optimistic schedule and the same of succeeding task under pessimistic schedule.

Nevertheless, this operation is simultaneously executed with step 06 where ∝ levels are utilized to aggregate the tasks to form optimistic and pessimistic schedules, following which step membership function of fuzzy Guntt bar is derived at given ∝ level.

5.8 Step 06: Defuzzification of the aggregated results

The fuzzy sets created by ∝-cuts (where the membership values disperse from zero to ∝) under step 05 for fuzzy Guntt bar to represent the existence of task performances are determined forming minimum violations to subsethood with the fuzzy intersections between their respective resource availabilities and balance project duration [$\mu_{RaRT(A) \cap BpdRT(A)}(t)$]. Microsoft Excel Solver add-in tool for optimization and equation solving is utilized to obtain the optimized preliminary solution for the case study.

The Solver add-in tool provides the optimized results generating levels of ∝-cuts by minimizing supersethoods (i.e. maximizing the subsethood) of fuzzy Guntt bar distributions forming with $\mu_{RaRT(A) \cap BpdRT(A)}(t)$. This operation is done adhering to the constraints developed under proposal and giving preliminary convergence considering the non-linearity towards ∝ = 0.5 as the mid point.

5.8.1 Results of the optimized preliminary solution

Preliminary solution for the case study is attained within the established constraints, ∝-levels appropriate for each recurring task as the decision variables and sum of violations to fuzzy subsethood as the objective function to be minimized. Hence, the minimized solution for violations to fuzzy subsethood that fuzzy Guntt bar [$\mu_{GunttRT(A)}(t)$] possesses against the fuzzy intersection between resource availability and balance project duration [$\mu_{RaRT(A)}(t) \cap \mu_{BpdRT(A)}(t)$] executes simultaneous computations deriving ∝-level for each recurring task within the established constraints.

5.8.1.1 Violations to fuzzy subsethood

The optimized solution generating minimum violations to subsethood derives aggregated violations as 0.0132. The results are graphically illustrated with figure 5.2 where the delineation enlightens the positioning of fuzzy Guntt bar [$\mu_{GunttRT(A)}(t)$] pertaining to each recurring task within the superset of intersection between fuzzy resource availability and balance project duration [$\mu_{RaRT(A)}(t) \cap \mu_{BpdRT(A)}(t)$] as appropriate.

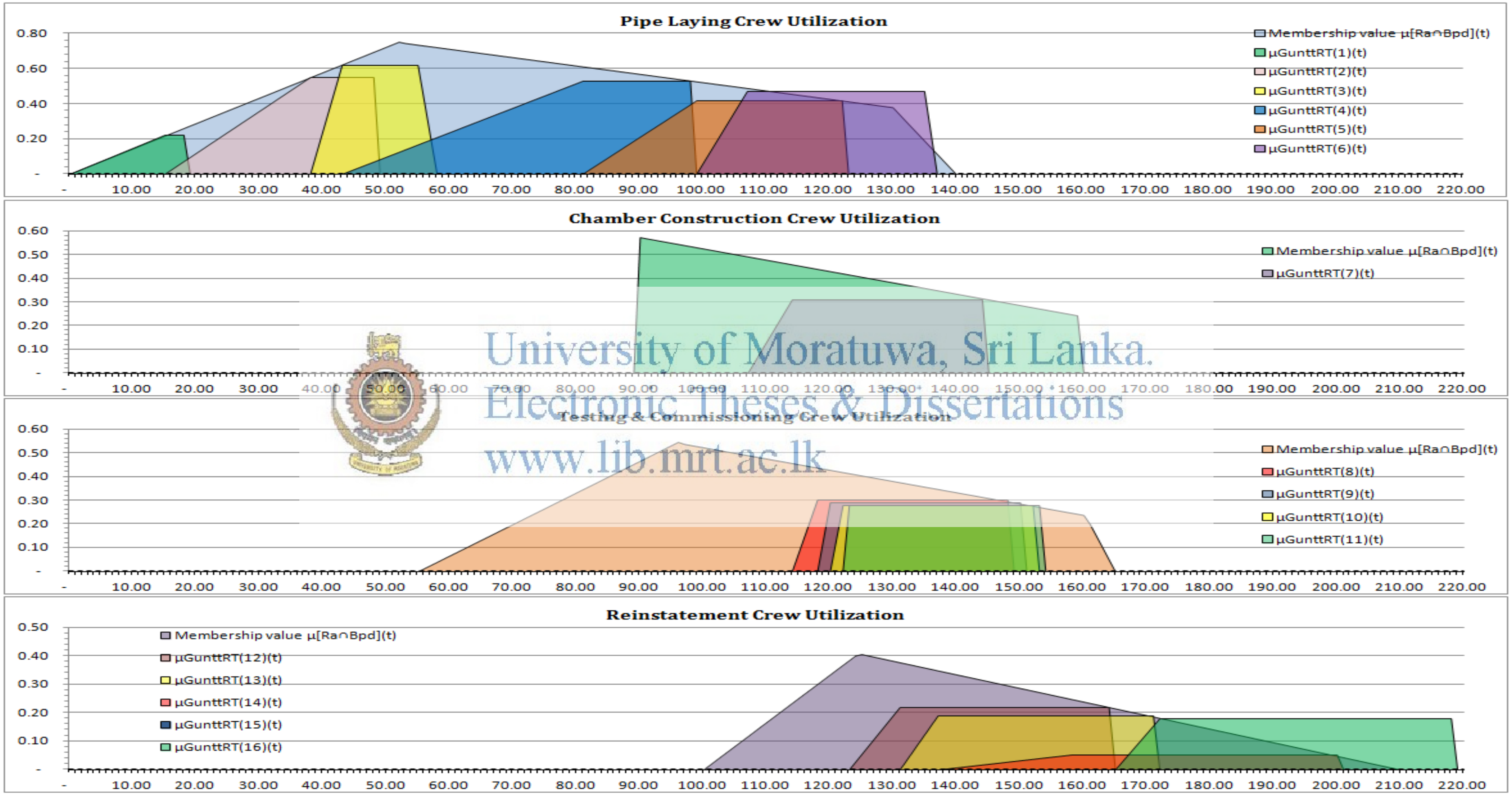


Figure 5.2: Positioning of subsets within supersets and minimized violations to fuzzy subsethood

5.8.1.2 Optimized fuzzy Guntt bar distribution

Fuzzy Guntt bar distributions for the recurring tasks are obtained following the derivation of optimized ∝-levels and respective optimistic and pessimistic task completion times. Figure 5.3 elaborates optimistic and pessimistic schedule times for the critical chain determined by optimistic and pessimistic task duration estimates and precedence relationships. Figure 5.4 depicts the fuzzy Guntt bar distributions for all recurring tasks comprising the critical chain.

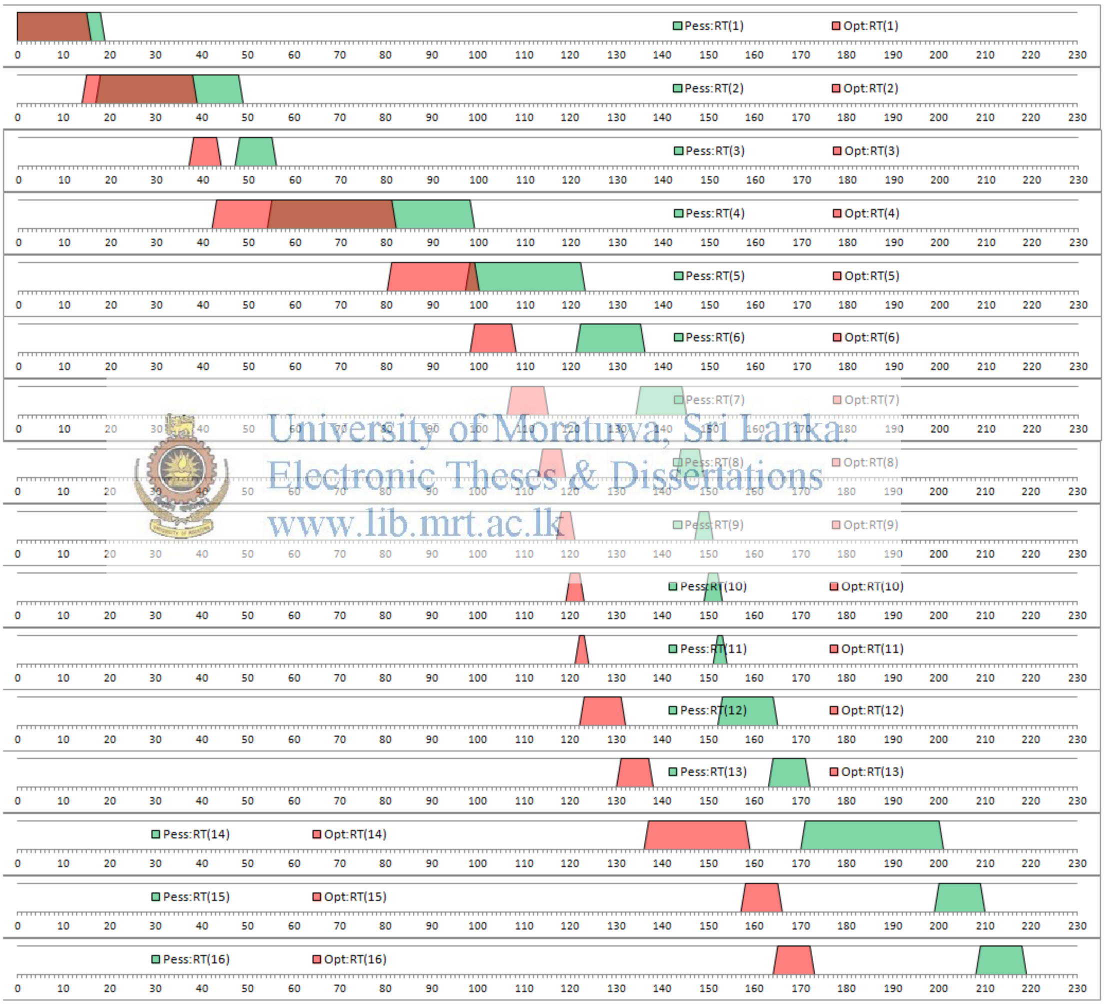


Figure 5.3: Optimistic and pessimistic scheduling times determined for the case study

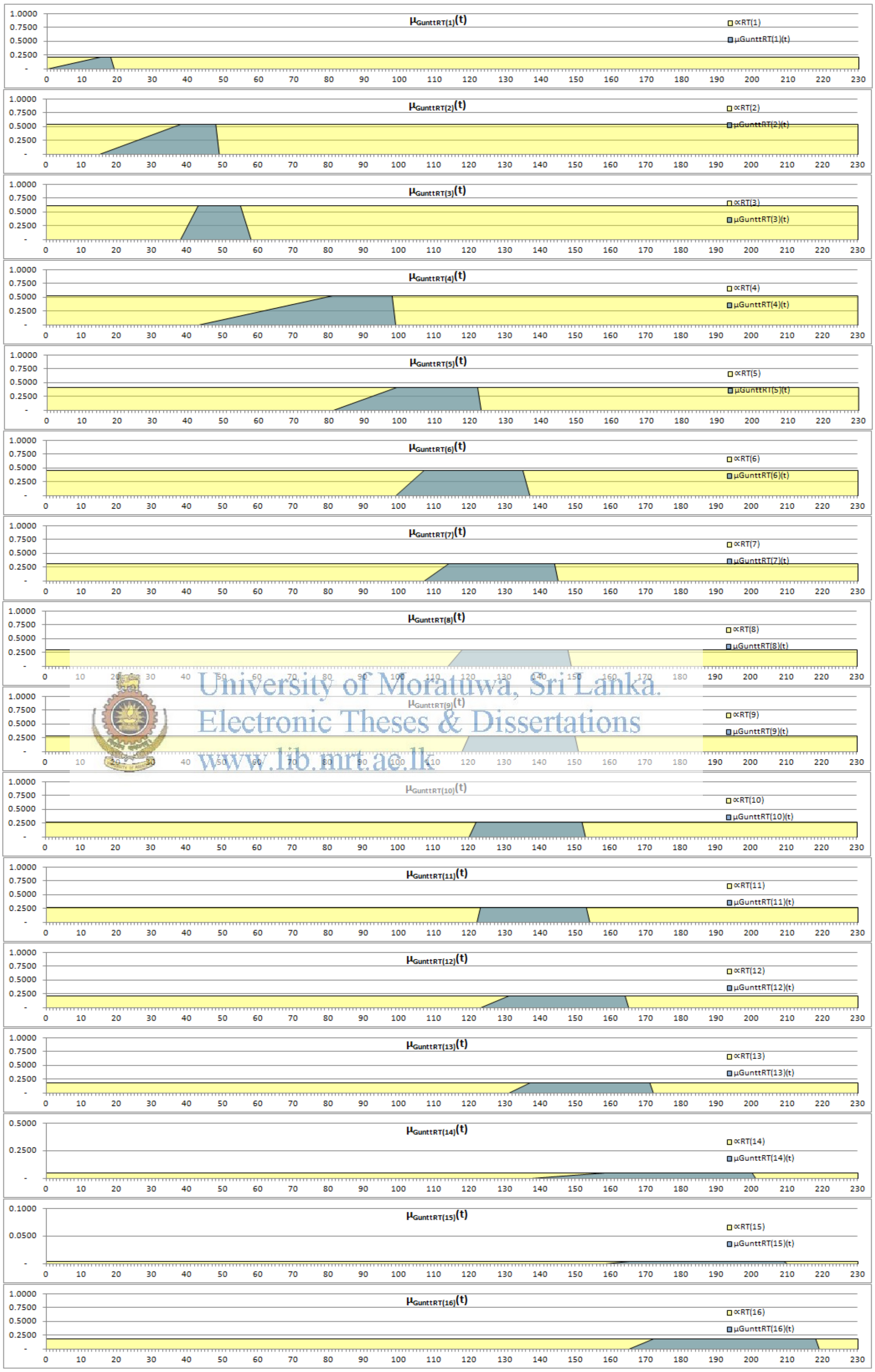
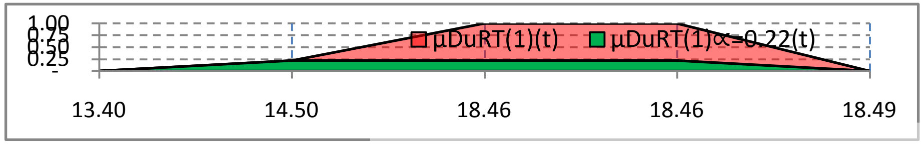
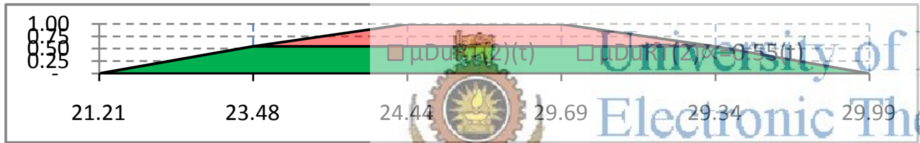
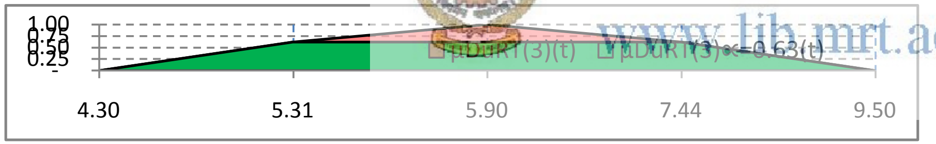
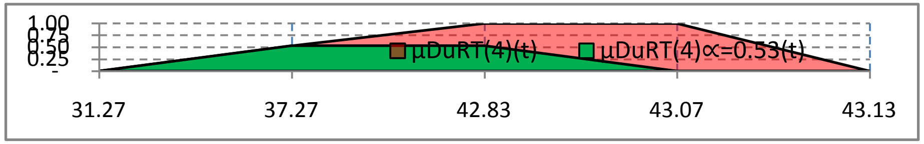
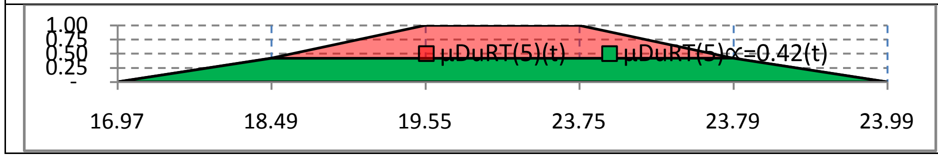


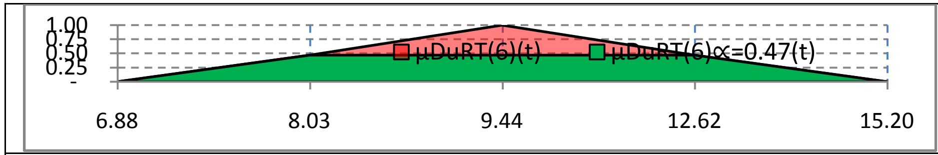
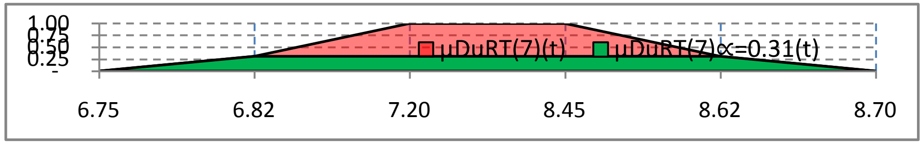
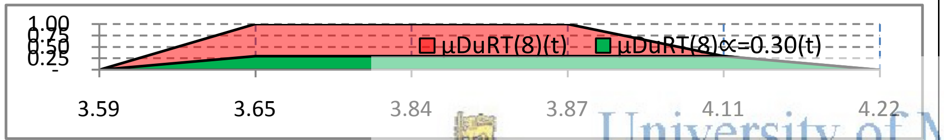
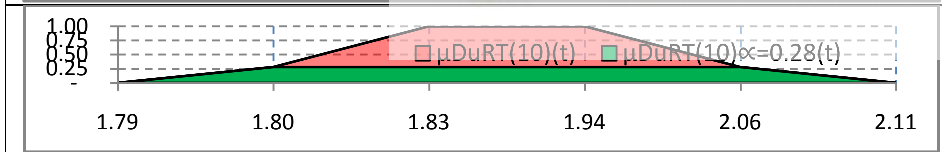
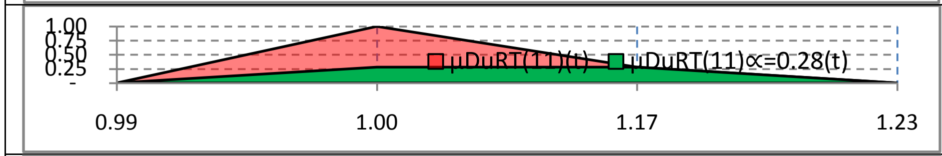
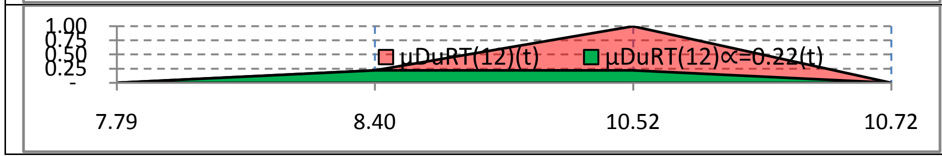
Figure 5.4: Fuzzy Guntt bar distributions for recurring tasks under critical chain

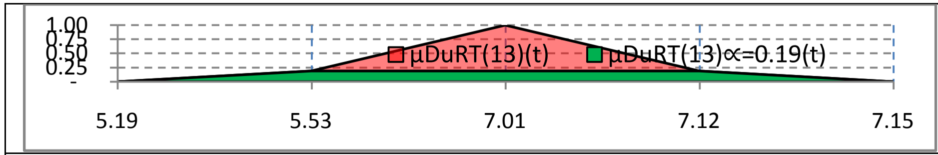
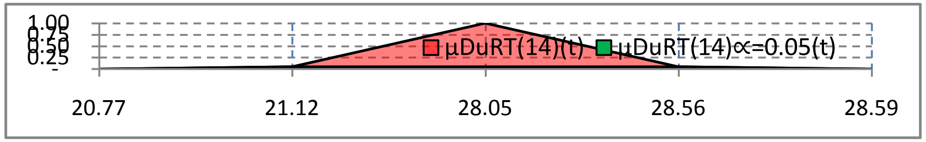
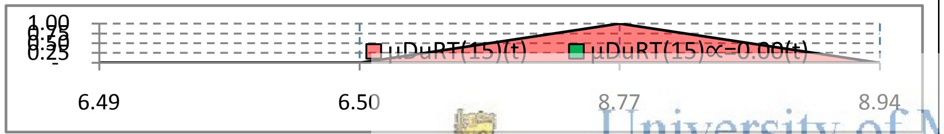
5.8.1.3 Contribution to project buffer by each recurring task

Along with the optimized result of minimum violation to fuzzy subsethood, the determined levels of ∝-cuts for each recurring task generate optimistic and pessimistic schedule durations. The difference between the optimistic and pessimistic durations for the task determined by ∝-level derives its contribution to the project buffer. Table 5.10 exhibits the calculations pertaining to the buffer contribution of each recurring task.

Table 5.10: Project buffer contribution by each individual task

$\mu_{DuRT(A)\propto}(t)$	TraFN[DuRT(A)] at ∝=1.0	TraFN[DuRT(A)] at determined ∝ level	Buffer Contribution (Days)
	{(13.40, 0.00), (18.46, 1.00), (18.46, 1.00), (18.49, 0.00)}	{(13.40, 0.00), (14.5, 0.22), (18.46, 0.22), (18.49, 0.00)}	3.00
	{(21.21, 0.00), (24.44, 1.00), (29.69, 1.00), (29.99, 0.00)}	{(21.21, 0.00), (23.48, 0.55), (29.34, 0.55), (29.99, 0.00)}	6.00
	{(4.30, 0.00), (5.90, 1.00), (5.90, 1.00), (9.50, 0.00)}	{(4.30, 0.00), (5.31, 0.63), (7.44, 0.63), (9.50, 0.00)}	2.00
	{(31.27, 0.00), (42.83, 1.00), (43.07, 1.00), (43.13, 0.00)}	{(31.27, 0.00), (37.27, 0.53), (42.83, 0.53), (43.07, 0.00)}	6.00
	{(16.97, 0.00), (19.55, 1.00), (23.75, 1.00), (23.99, 0.00)}	{(16.97, 0.00), (18.49, 0.42), (23.79, 0.42), (23.99, 0.00)}	6.00

	{(6.88, 0.00), (9.44, 1.00), (9.44, 1.00), (15.20, 0.00)}	{(6.88, 0.00), (8.03, 0.47), (12.62, 0.47), (15.20, 0.00)}	5.00
	{(6.75, 0.00), (7.20, 1.00), (8.45, 1.00), (8.70, 0.00)}	{(6.75, 0.00), (6.82, 0.31), (8.62, 0.31), (8.70, 0.00)}	2.00
	{(3.59, 0.00), (3.65, 1.00), (3.87, 1.00), (4.22, 0.00)}	{(3.59, 0.00), (3.65, 0.30), (4.11, 0.30), (4.22, 0.00)}	0.00
	{(1.98, 0.00), (2.00, 1.00), (2.00, 1.00), (2.46, 0.00)}	{(1.98, 0.00), (1.99, 0.29), (2.00, 0.29), (2.46, 0.00)}	0.00
	{(1.79, 0.00), (1.83, 1.00), (1.94, 1.00), (2.11, 0.00)}	{(1.79, 0.00), (1.80, 0.28), (2.06, 0.28), (2.11, 0.00)}	0.00
	{(0.99, 0.00), (1.00, 1.00), (1.00, 1.00), (1.23, 0.00)}	{(0.99, 0.00), (1.00, 0.28), (1.17, 0.28), (1.23, 0.00)}	0.00
	{(7.79, 0.00), (10.52, 1.00), (10.52, 1.00), (10.72, 0.00)}	{(7.79, 0.00), (8.40, 0.22), (10.52, 0.22), (10.72, 0.00)}	3.00

	{(5.19, 0.00), (7.01, 1.00), (7.01, 1.00), (7.15, 0.00)}	{(5.19, 0.00), (5.53, 0.19), (7.12, 0.19), (7.15, 0.00)}	1.00
	{(20.77, 0.00), (28.05, 1.00), (28.05, 1.00), (28.59, 0.00)}	{(20.77, 0.00), (21.12, 0.05), (28.56, 0.05), (28.59, 0.00)}	8.00
	{(6.49, 0.00), (8.77, 1.00), (8.77, 1.00), (8.94, 0.00)}	{(6.49, 0.00), (6.50, 0.00), (8.77, 0.00), (8.94, 0.00)}	2.00
	{(6.49, 0.00), (8.77, 1.00), (8.77, 1.00), (8.94, 0.00)}	{(6.49, 0.00), (6.90, 0.18), (8.91, 0.18), (8.94, 0.00)}	2.00

5.8.1.4 Preliminary solution; determined size of project buffer for the case study

The optimized solution generates optimistic project duration as 172 days and pessimistic duration as 218 day. Therefore it is evident that, pessimistic project duration is longer than the project duration i.e. 210 days constituted under project formalities. This is reasoned due to the absence of stipulated project duration as a constraint for the optimistic solution. The study found that, feasible solution being unviable in the presence the stipulated project duration as a constraint. Therefore, the project buffer determined for the case study under the proposed FIS approach is 38 days (210 days – 172 days).

Figure 5.5 elaborates the schedule of project critical chain for the case study where the timing of tasks exhibit optimistic scheduling times as computed under the proposed buffer sizing approach.

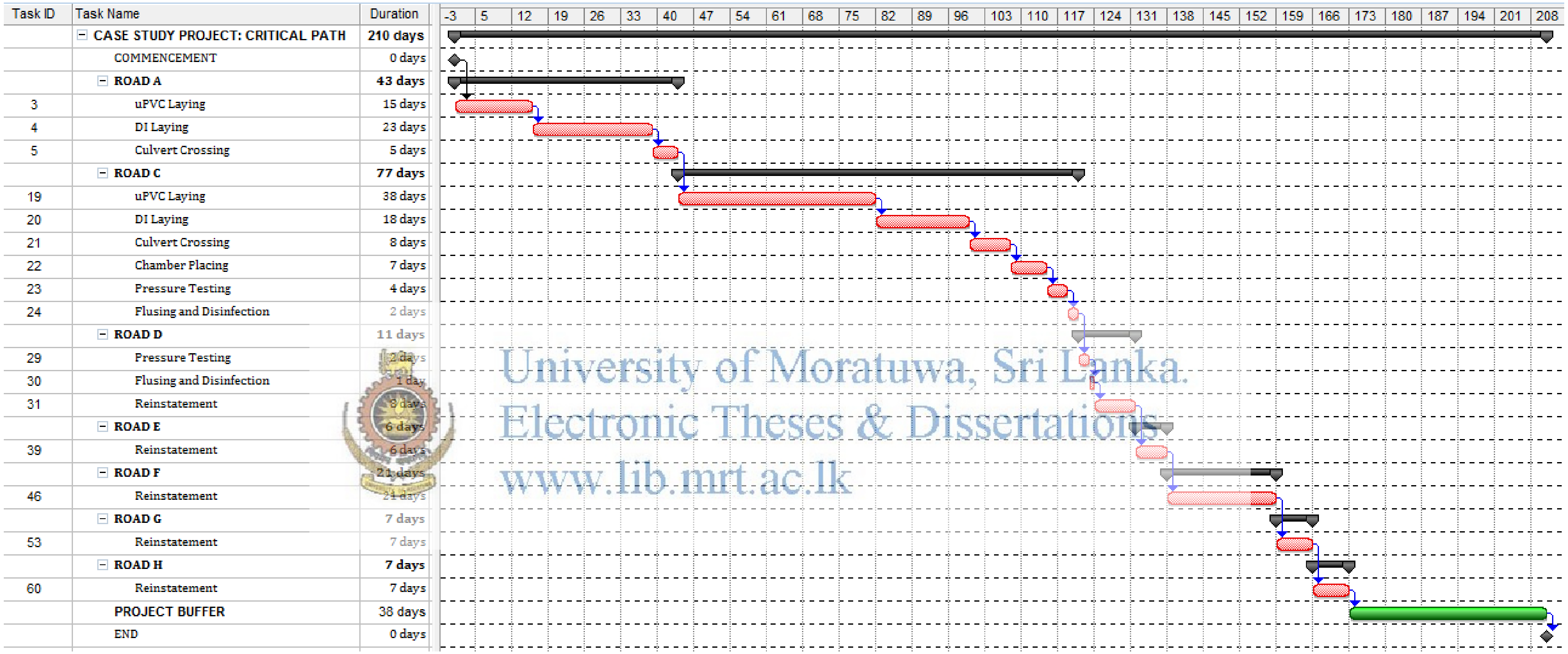


Figure 5.5: Schedule of critical chain with added project buffer under the proposed method of sizing

5.9 Summary

The chapter verifies the practical implementation of the approach proposed for buffer sizing under the preceding chapter.

The case study project is acquainted and statistics for the duration estimates for inbuilt recurring tasks are drawn referring a sample of project schedules with identified list of recurring activities.

The steps outlined in presiding chapter are implemented for the figurative observations under the statistics and for the tailor made sequencing of recurring tasks arrayed in the critical chain of the case study project schedule.

As delineated in the proposal, defuzzification by deriving relevant α-cut levels for each recurring task is obtained whilst optimizing fuzzy subsethood by resorting for non-liner programming. The chapter thus consummates with the optimistic preliminary solution evoked by Microsoft Excel Solver add-in programme for the size of project buffer for the case study.

CHAPTER 06

CONCLUSION AND RECOMMENDATIONS

6.1 Conclusions of the study

The pioneering canvassing of intellectual views prior the study divulged; though CCPM not being implemented in water distribution projects as a project management strategy, its mere substance of buffer concept is yet apt to be experimented. This is triggered by the fact; buffers are used to be housed intangibly and in clandestine within the project schedules even unknown to the schedulers themselves. Nevertheless, it is conceived that buffers are durations which are strategically sized and added to the schedules. Hence, they are not perceptive for being investigated for their certainty of magnitude. Therefore, the current study converges and contains its search lights only towards the concept of project buffer and portrays a new strategic way of its sizing.

Reduction in overall project duration is not aimed under the proposed approach for buffer sizing as CCPM exploits. Instead the study ushers in a new strategic way of sizing project buffer by optimizing schedule performances where balance project duration and resource availability impede. Nevertheless, a figurative comparison is found infeasible between the proposed method and the existing method of buffer sizing. This is due to the fact, buffer being an intangible entity within a project schedule leaves no room to be brought under the microscope of statistical sampling. Hence, the study concludes by delving in to the distinguishable properties of the methods comprising their respective pros and cons.

The study gives up its context to the statistical law of aggregation and central limit theorems upon which CCPM arbitrary apportionment for buffer sizing is conceptualized. The literature signifies that, right long skewed nature of the typical task duration distribution dissents for 50% apportionment of optimistic schedule duration and is further degraded as being prone to disrupt the resource contention if not task scheduling is brought under the lights of empirical vigilance. The proposed fuzzy set theory based method is apart from such philosophies based on "randomness" in the formation of its ideology. Contrary to randomness, the new approach denominates a membership value upon the time axis i.e. universe of discourse being considered, to represent its fuzzy variables. Thus formulated fuzzy sets then undergo operations such as intersections, compliments, subsets, supersets etc. under the set theory of mathematics.

Similarly as, common cause variation being the CCPM focus, the proposed method relegates factor analysis as many other heuristic procedures do advocate. Alternatively the new method yields a "cause and effect" base approach under dynamic scheduling considerations. The resultant level of $\propto$-cut optimizing fuzzy subsethood is caused by positioning fuzzy Guntt bar (subset) within the superset of intersection between fuzzy balance project duration and resource availability. For all task related (fuzzy Guntt bar) and project related (fuzzy balance project duration and fuzzy resource availability) causes, the effect is optimized to the desired orientation of the subsethood. The method proposed in CCPM only determines its aforementioned fractioning between task scheduling time and buffer contribution devoid of a cognizance to an optimized result.

The convenience of handling incomplete and imprecise data with FIS buttressed in creating Fuzzy set for the potential for student syndrome in task duration estimate. The vested uncertainty propagation technique permits straight lines instead of curves to the known direction of rise or fall of the membership value. Hence, triangular fuzzy sets are formed to represent the potential for student syndrome of task duration estimates devoting membership values zero to the lowest task duration and one to the highest task duration. It is literally known that, membership value thus varies but the exact trend of varying its inclination is not infused.

The proposed method allows semantic of the natural language (linguistic) statements to be modeled. Therefore, the outcomes of optimistic and pessimistic durations of tasks are enabled to the extent of concession provided by the project related fuzzy variables. Hence, the new approach performs its buffer sizing conforming to the principles abide by the linguistic statements "the more the resource availability and balance project duration are available the larger the optimistic duration (optimistic duration is more confident) and smaller the buffer contribution", and vise versa "the smaller the resource availability and balance project duration the shorter the optimistic duration and the larger the buffer contribution". The obtained result is always optimized to the desired orientation of linguistic statements with the aid of ∝-cut arithmetic operations and by paying recourse to operational research technique; non linear programming. The optimization algorithm selects between various available ∝-levels and filters the exact solution for the optimized result.

CCPM acquaints critical chain as the longest chain of activities convoluted with task dependencies and resource contention. The new approach demarcates the critical chain and address the subsequent buffer sizing subject to the identified resource dependencies considering the periods of resource availabilities. Nevertheless, the new approach does not necessitate resource leveling. Resource constrained project scheduling problem is optimally solved for the critical chain. Resource constraint is not considered for the feeding paths due to the drawn research limitations. Therefore, resource constraint is cardinally resolved exploiting the system constraint whilst feeding paths being subordination to the former, the resolution for their resource contention thus becomes a sideline out of drawn limitations to the study.

The optimistic task duration is supposed under the new approach to be aggressive whilst being reasonably long enough to achieve. This is satisfied via forming a ceiling to limit the level of ∝-cut. This maximum of ∝-level is the maximum of membership value between the fuzzy intersection between task duration estimate and respective potential for student syndrome.

The new approach enables computing revised buffer estimate taking in to consideration of balance quantities of the works under each task to be performed, balance project duration and the revised resource availabilities. Whereas the activity path merging is experienced, the project buffer shall be recalculated to the balance work load of the revised critical chain, balance project duration and updated resource availability for the tasks on revised critical chain. This supposedly more pragmatic for project scheduling in water distribution projects since activity path merging is not to be left behind in oblivion.

6.2 The essence of pragmatism evoked from the case study

Project critical paths have not been limitations for over the samples collected to form task duration distributions. Sample statistics are chosen irrespective of their belonging to critical or feeding paths to evict the effect of the length of project duration (whether comparatively lengthier or shorter) being reflected in the task durations. This is further important for the sample statistics to construe common cause variations.

Figurative comparison of the lengths of project buffers computed based on the new method and CCPM method are inappropriate. As in fact project buffers are strategically sized and located, figurative comparisons drive no comprehensions as buffer penetrations could not be statistically tracked down. Qualitatively assessed strategies of buffer sizing referring to their pros and cons gain more empirical wealth in comparison.

Formation of fuzzy sets as codified in the new approach does not convince being a precise method. Fuzzy sets appear as constituent of inferences and approximations as they are transformed from random sets to fuzzy sets.

The new approach demonstrates its operation via trapezoidal fuzzy sets as in fact trapezoidal fuzzy sets are reported to be effective in representing data ranges in dispersions (ex. durations) than points of convergences. Nevertheless, giving effect to the semantic of linguistic variables drawn in sizing project buffer triangular configurations appears more rationale. This is because, the new approach comprise minimum of a compulsive non-zero value for the buffer with trapezoidal configuration, which can be reformed by transforming to a triangular configuration. With triangular configuration there will be a minimum of zero buffer at the highest level of $\propto$, where in fact resource availability and balance project duration optimally sanction for a confident optimistic duration devoid of any buffer contribution for the task.

Since resource availability is a volatile parameter, the new approach invariably undergoes the need for the hardship of revising perpetually at shorter intervals. The case study presented in this research elucidates the derivation of preliminary solution considering the resource availability ascertained at the initiation of the project.

In its optimization algorithm the case study failed to establish the formal project duration as a constraint. Hence the buffer contribution by the ending task of the critical chain incorporates an error. This drawback is emerged by the unavailability of an optimized solution whilst project duration being a constraint. Nevertheless, this does not interpret that optimization operation is an outright failure to the expected orientation of optimizing. The optimistic task duration at this juncture does not exhibit a failure since the resource availability constraints are optimally satisfied. Despite project duration failing to become a constraint to the optimistic solution, the optimistic solution is found viable, where of cause pessimistic project duration is longer than the formal project duration.

The optimization algorithm performed in Microsoft Excel is of an inherent impotence of implementing its computations for all the possible positioning of subsets within the supersets. Hence the algorithm to encapsulate violations to fuzzy subsethood does not encompass all possible occurrences. This is enabled by the lack of eligibility in Microsoft Excel to formulate and programme the suggested optimization algorithm.

6.3 Recommendations and further researches

Project feeding paths being out of contemplations under the research, their minimum path lengths following the curtailment of critical task duration estimates in to their respective optimistic estimates are not guaranteed. Hence, further studies shall be drawn to size the feeding buffers with realistic feeding path duration estimates, along with the proposed method of project buffer sizing.

The proposed optimization algorithm operated in Microsoft Excel spread sheet environment fails to encounter all possible events of positioning fuzzy subsets within their supersets. Furthermore, necessity of obtaining revisions to the size project buffer along with the activity path penetrations, dynamics of balance workload, outstanding project duration and volatilities in resource availability requires more potential in programming over the facilities provided by Microsoft Excel. This impotence should be streamlined via being recourse to a more sophisticated programming tool.

References

- Ahmad, S. U., & Akther, T. (2009). A computational method for fuzzy arithmetic operations. *Daffodil International University Journal of Science and Technology, 4*(1), 18-22.

- Alola, A., Alola, V., & Tunay, M. (2013). Analysis of possibility theory for reasoning under uncertainty. *International Journal of Statistics and Probability, 2*(2013), 12-23.

- Ash, R. B. (2008). *Basic probability theory.* Mineola, New York: Dover Publications.

- Aryanezhad, M.B., Ashtiani, B., & Fallah, M. (2010). Critical chain project scheduling: utilizing uncertainty for buffer sizing. *IJRRAS, 3*(3), 280-289.

- Barnes, R., Dvir, D., & Raz, T. (2003). A critical look at critical chain project management. *Project Management Journal, 34*(4), 24-32.

- Bevilacqua, M., Ciarapica, F.E., & Giacchetta, G. (2008). Critical chain and risk analysis applied to high risk industry maintenance: a case study. *International Journal of Project Management, 27*(2009), 419-432.

- Dhar, M. (2012). A note on subsethood measure of fuzzy sets. *International Journal of Energy, Information and Communication. 3*(3), 55-62.

- Dhar, M. (2013). A note on entropy subsethood relationship. *International Journal of Intelligent Systems and Applications in Engineering, 1*(3), 44-46.

- Dubois, D. (2006). Possibility theory and statistical reasoning. *Computational Statistics and Data Analysis Journal, 51*(1), 47-69.

- Dubois, D., & Prade, H. (2011). Possibility theory and its application: where do we stand? *Mathware and Soft computing, 18*(1), 18-31.

- Eksioglu, S.D., Rom, W.O., & Tukel, O.I. (2006). An investigation of buffer sizing techniques in critical chain scheduling. *European Journal of Operational Research, 178*(2006), 401-416.

- Elmaghraby, S.E., Herroelen, W.S., & Leus, R. (2002). Note on the paper 'Resource-constrained project management using enhanced theory of constraint' by Wei et al. *International Journal of Project Management, 21*(2003), 301-305.

- Geekie, A., & Steyn, H. (2008). Buffer sizing for the critical chain project management method. *South African Journal of Industrial Engineering, 19*(1), 73-88.

- Goto, H.,Takahashi, H., Takei, Y., & Truc, N.T.N. (2012). Buffer management method for multiple projects in the CCPM-MPL representation. *Industrial Engineering & Management Systems, 11*(4), 397-405.

- Hait, A., & Masmoudi, M. (2012). Fuzzy uncertainty modeling for project planning; application to helicopter maintenance. *International Journal of Production Research, 50*(13), 3594-3611.

- Hait, A., & Masmoudi, M. (2013). Project scheduling under uncertainty using fuzzy modeling and solving technique. *Engineering Applications of Artificial Intelligence, 26*(2013). 135-149.

- Howell, G., Hsiang, S.M., Liu, M., Russell, M. (2012) Causes of time buffer in construction project task durations. In *Proceedings for the 20th annual conference of the International Group for Lean Construction,* Retrieved from http://www.iglc20.sdsu.edu/papers/wp_content/uploads/2012/07/88%20p%20126.pdf

- Jan, S., & Ping, H.S. (2006). Construction project buffer management in scheduling planning and control. In *ISARC2006,* (pp. 858-863). Retrieved from http://www.iaarc.org/publications/fulltext/isarc2006-00111_200608141302,pdf

- Kokoskie, G. (2001). A comparison of critical chain project management (CCPM) buffer sizing techniques. In *SYST798 Research Project,* Retrieved from http://www.dtic.mil/dtic/tr/fulltext/u2/a400313.pdf

- Leach, L.P. (1999). Critical chain project management improves project performance. *Project Management Journal, 30*(2), 39-51.

- Leach, L.P. (2005). *Critical chain project management* (2nd ed.). Norwood: Artech house.

- Leavengood, J., & Reeb, J. (2002). Transportation problem: a special case for linear programming problems. In *Operations Research,* Retrieved from https://catalog.extension.oregonstate.edu/sites/catalog.extension.oregonstate.edu/files/project/pdf/em8779.pdf

- Liu, P., Tsai, Y., & Wei, C. (2000). Resource constraint project management using enhanced theory of constraint. *International Journal of Project Management, 20*(2002), 561-567.

- Long, L.D., & Ohsato, A. (2007). Fuzzy critical chain method for project scheduling under resource constraints and uncertainty. *International Journal of Project Management, 26*(2008), 688-698.

- *Oxford dictionary (7th ed.). (2000). Hinsdale, IL: Penguin Press.*

- Radovilsky, Z. D. (1993). A quantitative approach to estimate the size of the time buffer in the theory of constraints. *International Journal of Production economics. 55*(1998), 113-119.

- Rand, G. K. (2000). Critical chain: the theory of constraints applied to project management. *International Journal of Project Management. 18*(2000), 173-177.

- Sarkar, S. (2012). Transition from critical path to critical chain: a case research analysis. *International Journal of Scientific & Engineering Research, 3*(9), 1-10.

- Steyn, H. (2000). An investigation in to the fundamentals of critical chain project scheduling. *International Journal of Project Management, 19*(2000), 363-369.

- Trietsch, D. (2004). The effect of systemic errors on optimal project buffers. *International Journal of Project Management, 23*(2005), 267-274.

- Tsoukalas, L. H., & Uhrig, E. R. (1997). *Fuzzy and neural approaches in engineering*. New York: John Wiley & Sons Inc.

- Wierman, M. J. (2010). *An introduction to the mathematics of uncertainty*. Omaha: Creighton University.

- Zadeh, A. L. (1978). Fuzzy sets as a basis for theory of possibility. *Fuzzy Sets and Systems Journal. 1*(1), 3-28.

Appendix-01:

Initial programme and works breakdown structure of the case study project