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# JERK CONTROLLED JOINT SPACE TRAJECTORY PLANNING METHOD USING 5-3-5 SPLINE TRAJECTORY FOR INDUSTRIAL ROBOT MANIPULATORS 


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Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree Master of Science in Electronics and Automation.

Department of Electronic and Telecommunication Engineering


## DECLARATION

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#### Abstract

In robot manipulator trajectory planning, jerk controlled trajectories are desirable for their amenability in path tracking, vibration suppression and reducing of manipulator wear. Bounded and continuous jerk trajectories narrows the jerk profile within known limits, eliminating discontinuities and spontaneous rising of jerk, which are otherwise undesired. In this research, a new method for generating a bounded and continuous jerk trajectory in joint space is implemented and tested. Spline functions are a common method for interpolating a set of via points for generating robot manipulator trajectories. The method introduced in this research is a novel and robust spline interpolation algorithm, interpolating a $5^{\text {th }}$ order, $3^{\text {rd }}$ order and $5^{\text {th }}$ order piecewise polynomial ( $5-3-5$ spline), which can generate point-to-point trajectories as well as trajectories with via points. Generated trajectory has continuously differentiable profiles for position, velocity and acceleration and has a start and end zero-bounded, continuous jerk profile. The algorithm allows the user to independently define the position, velocity, acceleration and jerk values at both start and end points. At via points, the continuity up to $5^{\text {th }}$ time derivative is maintained, and the user can define via point position and velocity. Also it allows the user to define interpolation time intervals between every polynomial segment. Along with the bounded and continuous jerk profile, these adjustable parameters in the 5-3-5 spline algorithm results an effective and robust trajectory generating method. Trajectories generated using the $5-3-5$ spline algorithm was simulated and tested successfully on DENSO VP6 robot arm. The experiments demonstrated and proved the robustness and applicability of the new trajectory generating method in both point-to-point and via point motions


Key words: Joint space trajectory planning, controlled jerk, continuous acceleration, piecewise polynomials, 5-3-5 spline trajectory, point-to-point motion, motion with via points

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Appendix-A. Classification of Industrial Robots according to $I S O$ 8373:1994, Manipulating industrial robots Vocabulary [5]

